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The effect of path cut on Somigliana ring dislocation elastic fields

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Abstract

In this paper we look at ring dislocations (circular loops) in an infinite isotropic full-space. The dislocation direction is either axial or radial. Unlike dislocations in plane analysis the path cut has a significant effect on the elastic fields. Solutions for the dislocations are given for a variety of path cuts with closed form expressions for the displacement and stress fields. When considered alone these dislocations do not obey Frank's rule; these anomalies and other fundamental properties are discussed.

Keywords: Dislocations; Somigliana; Axisymmetric

1. Introduction

Dislocations were hypothesised at the end of the 19th century and subsequently (1930s) proposed to explain the difference between the Frenkel calculation for the theoretical strength of a crystalline solid, and measured values. Their existence was, of course, subsequently verified experimentally: dislocations in crystalline solids are of the Volterra kind (Christian and Crocker, 1980), and, in solid mechanics terms, may be formed by taking a cut along a surface from the dislocation line to infinity, displacing the two surfaces everywhere by a constant amount, the Burgers vector, adding or removing material as necessary, and glueing the cut surface back together. Volterra dislocations have, amongst other properties, the features that the Burgers vector is constant along the path cut and that the state of stress induced is *independent* of the location of the surfaces (i.e. it is path-cut independent). The stress induced by the procedure varies like $1/r$, where r is a coordinate measured from the dislocation line and orthogonal to it, and hence has no long-range resultant. This last property, in particular, makes dislocations attractive in another field entirely: they can be used as strain nuclei to perturb the state of stress in a solid in a controlled way, possibly to model plastic flow (Blomerus and Hills, 1998), or to solve crack problems. The latter procedure was developed extensively by Dundurs, Keer, Comninou and others in the 1970s, and some of the techniques were summarised by Hills et al. (1996). In this paper we wish to

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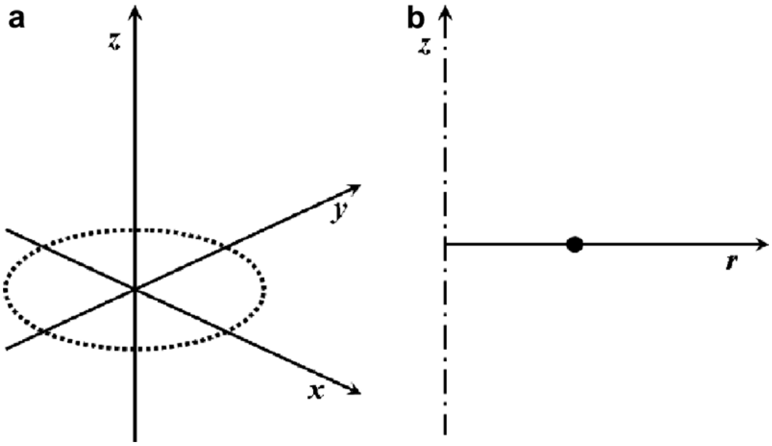

Fig. 1. Ring dislocation and axes.

examine one geometry of a dislocation loop – when it is a circle – and to examine, in particular, the properties of the solution when the Volterra assumptions do not hold.

Fig. 1(a) shows an axis set present in an infinite space, with the dislocation loop, of radius a, lying in the $x-y$ plane. Two types of Volterra type edge dislocation can be identified: the climb or prismatic loop, when the Burgers vector lies in the z-direction, and the glide loop when it lies in the $x-y$ plane. The literature includes solutions for climb loops in an infinite space (Korsunsky, 1996a; Kroupa, 1960; Salamon and Comninou, 1979) and a half-space (Korsunsky, 1996b; Salamon and Dundurs, 1971). The glide loop is less comprehensively covered, but solutions for an infinite and half spaces are given for example in Salamon and Dundurs (1977). These solutions are of value in studying crystalline defects, but the glide loop is of very limited value as a strain nucleus, as it is not inherently axisymmetric. A much more useful strategy for axisymmetric crack problems is to develop a dislocation solution family based on the cylindrical coordinate set shown in Fig. 1(b). Clearly, the prismatic loop (Burgers vector b_z) is equally valuable here, and the 'twist loop', in which the Burgers vector of the dislocation is tangential to the dislocation line is very useful for torsional problems. In the latter case the Burgers vector varies in direction around the loop but there was little difficulty in developing a solution for both an infinite space and a half-space (Sackfield et al., 2002). The remaining dislocation solution needed is one for a radial Burgers vector (b_r). There are existing solutions by Korsunsky (1996a) and by Demir et al. (1992), but these make intrinsic assumptions about the path cuts used to form the dislocation, and these may or may not be consistent with the intended application. Because generalised Somigliana dislocations are being considered the state of stress induced *is* dependent on the path cut, and, as will be shown, there are further complications in their exploitation as strain nuclei. The object in the present paper is to provide a definitive solution for both kinds of axisymmetric ring dislocations, in an infinite space. Although there are pre-existing solutions, listed above, none of them has sought to emphasize the difficulties which arise when analysing the radial dislocation, and this is particularly important when dislocations are to be employed as strain kernels.

2. Dislocation definitions

A plane edge dislocation may be formed, in solid mechanics terms, either by making a cut along the glide plane and displacing the adjacent surfaces in shear by the Burgers vector, or by making a cut from the core perpendicular to the glide plane, and inserting a thin sheet, of the same material, of thickness equal to the Burgers vector. The stress field induced by each is identical, and so is the component of the displacement field perpendicular to the Burgers vector, to within an arbitrary constant. However the component of displacement parallel with the Burgers vector suffers a jump when the path cut is crossed, and hence differences do arise. When dislocations are used to model cracks the path cuts should be arranged to lie along the line of the crack,