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Steady penetration of a rigid cone into pressure-dependent plastic material

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Abstract

The objective of the present paper is to find a semi-analytical axisymmetric solution for steady penetration of a rigid cone into pressure-dependent plastic material obeying the double-shearing model. As expected, the solution is singular near the maximum friction surface. It is important to mention that the singularity is not due to the geometry of the problem but the friction law. The type of the singularity is the same as in plane-strain solutions based on the double-shearing model and in classical plasticity. This allows for calculating the strain rate intensity factor. The solution is illustrated by a numerical example.

Keywords: Penetration; Friction; Singular solution; Double-shearing model; Pressure-dependent plasticity

1. Introduction

Steady penetration of a rigid cone or wedge into a medium belongs to a group of classical plane-strain and axisymmetric problems in plasticity that also includes compression between parallel plates, flow through channels and other similar problems. The main assumption accepted in all these solutions is that the orientation of the principal stress depends on a single coordinate only. Solutions for plane-strain and axisymmetric penetration into various plastic media have been obtained by Fleck and Durban (1991), Durban and Rand (1991), Durban and Fleck (1992) and Durban (1999). These studies have focused on

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the singular behavior of solutions in the vicinity of the apex. In the present paper steady penetration of a rigid cone with the rough surface into rigid/plastic material obeying the double shearing model of pressure-dependent plasticity is considered. A solution to an analogous plane-strain problem has been obtained by Alexandrov and Lyamina (in press). The double-shearing model for granular materials under plane-strain conditions has been proposed by Spencer (1964). The systems of equations for axisymmetric and three-dimensional deformations have been given in Spencer (1982). The model is based on the Coulomb–Mohr yield condition and the assumption that deformation occurs by shear on the characteristic curves of stress equations consisting of the yield condition and the equilibrium equations. It does not include the normality rule but includes the incompressibility equation. Another important property of the model is that the stress characteristics coincide with the velocity characteristics. A number of analytical and semi-analytical plane-strain solutions based on the double-shearing model generalizing the corresponding solutions in classical plasticity have been obtained by Pemberton (1965), Marshall (1967), Spencer (1982), Alexandrov and Lyamina (2003). All of these solutions lead to singular velocity fields in the vicinity of friction surfaces where the maximum friction law is adopted. It is important to note that this type of singularity is quite different from that emphasized in Fleck and Durban (1991), Durban and Rand (1991), Durban and Fleck (1992), Durban (1999) and Papanastasiou et al. (2003). The latter is caused by the geometry of the problem and occurs in the vicinity of the apex, whereas the former is caused by the maximum friction law and occurs in the vicinity of the friction surface. The maximum friction law postulates that a characteristic direction (in the case of the double shearing model the characteristic directions for stress and velocity equations coincide) is tangent to the friction surface. In the case of plane-strain deformation, it has been shown in Alexandrov and Lyamina (2002) that the singular solutions occur near friction surfaces where an envelope of characteristics coincides with such a surface. There is no general result on singular solutions for axisymmetric flows of materials obeying the double-shearing model. However, a solution for flow through an infinite converging channel shortly described in Spencer (1982) shows that this particular velocity field is singular. Other solutions for axisymmetric deformation of materials obeying the double-shearing model are given in Spencer (1983, 1984, 1986). However, these solutions do not involve the maximum friction law. In particular, in Spencer (1984) steady penetration of a rigid cone with a frictionless wall has been studied. In the present paper, the same problem with friction is solved, assuming the maximum friction law at the cone surface. An essential difference between these formulations is that in Spencer (1984) a face regime on the yield surface occurs whereas the present solution requires an edge regime. The latter solution shows that the velocity field is singular near the maximum friction surface and the type of singularity is the same as in plane-strain solutions based on the double-shearing model (Alexandrov and Lyamina, 2002) and in arbitrary flows of classical plasticity (Alexandrov and Richmond, 2001).

2. Statement of the problem

A rigid cone is penetrating an incompressible pressure-dependent plastic solid under axisymmetric conditions. End effects are neglected. Without loss of generality, it is possible to assume that the cone is motionless whereas the material moves with a velocity U as shown in Fig. 1. It is convenient to introduce a spherical coordinate system $r\theta\varphi$ with its origin at the cone apex. Then, the surface of the cone is defined by the equation $\theta = \theta_0$. It is assumed that there exists a rigid/plastic boundary defined by the equation $\theta = \theta_p$. The value of θ_p should be found from the solution. The solid obeys the double-shearing model (Spencer, 1982). The model includes the Coulomb–Mohr yield condition. In the case of axisymmetric deformation, several regimes on this yield condition described in Spencer (1982) are possible. For the problem under consideration, the appropriate regime corresponds to point F (Fig. 2) and is defined by the following equations: