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Nonlinear stability of a shallow unsymmetrical heated orthotropic sandwich shell of double curvature with orthotropic core

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Abstract

Equations which govern the behavior of an elastic unsymmetrical, orthotropic sandwich shell of double curvature with orthotropic core having different elastic characteristics under uniform heating are derived. The face sheet may be of unequal thickness of different materials. However, a restriction that the radii of curvature of the shell elements be large compared with the overall thickness of the sandwich shell is imposed. The variational procedure has been used to obtain the five equations which govern the behavior of the heated orthotropic sandwich shell for the stability. In case of symmetry the equations resemble with those of Grigolyuk. Finally, the numerical results of a square or a rectangular simply supported curved plate section of a cylindrical shell under thermal loading have been computed and compared with other known results. The graphs have been drawn to show the effects of different sandwich material for immovable and movable edge conditions.

Keywords: Nonlinear differential equations; Orthotropic material; Heated sandwich shell; Stability

1. Introduction

The field of sandwich construction, while not new, has become quite important in recent years as a result of improvement in manufacturing techniques. It has long been recognized as an efficient method of obtaining a lightweight compression member, but the prohibited cost of construction has limited its use. However, as new manufacturing methods are now being developed which make the use of sandwiches economically feasible, the collection of more research data is becoming increasingly important.

The first significant contribution to an understanding of the behavior of sandwich shells was presented by Reissner (1950), who showed the effects of shear deformations and core compression that differentiate the

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sandwich theory from ordinary shell theory based on the Kirchhoff-Love assumption. Since then numerous papers have been published discussing analytical and experimental results of studies dealing with statically loaded cylindrical shells (Eringen, 1951; Stein and Mayers, 1952; Raville, 1954, 1955a,b; Wang and Desanto, 1955; Haft, 1955; Radkowski, 1957).

More recent investigations have extended the theory of sandwich shells to include doubly curved shells(Korolev, 1955; Grigolyuk, 1957, 1958a,b), fully plastic cores (Grigolyuk, 1958a,b), creep (Hoff et al., 1959), and free vibrations (Yu, 1960; Chu, 1961). Kamiya (1976), using Berger's (1955) approximations in large deflection analysis, has offered a new set of governing equations to study the nonlinear static behavior of sandwich plates. But his attempt has been restricted to plate geometry due to introduction of a correction factor. Although Berger's method has been applied to solve ordinary shallow shell problems, this method fails completely for movable edge conditions. Alwan (1964) and Nowanski and Ohnabe (1976) have also derived equations of sandwich plates and shells with orthotropic core for the analysis of large deflection. Recent investigations have also enriched the theory of sandwich shells by including doubly curved shells (Fulton, 1961; Bera, 1996).

The large amplitude vibration of thin elastic plates and stability analysis of heated sandwich plates and shells are also very important in modern design. Unfortunately in this field also not many works have been done so far. The only recent works of Kamiya (1978a,b), Ray et al. (1993), Dutta and Banerjee (1991), Banerjee (1981) and Bera et al. (1996, 1998, and 1999), in the large deflection analysis of heated sandwich plates and shells, can be located. The works of Librescu et al. (1997, 1998 and 2000), on nonlinear modeling of sandwich plates and shells are also noteworthy. But in Kamiya's works, following Berger's method, no result for movable edge condition is obtained.

The purpose of the present paper is to develop a simple and yet sufficiently accurate method for stability of heated orthotropic sandwich shells with orthotropic core and faces. In deriving the equations, the idea of Bera (1996), used in the case of isotropic symmetrical sandwich shell has been utilized with purpose and profit. It is interesting to note, however, that the equations obtained by Fulton (1961) , for the sandwich shells for symmetrical faces can be easily deduced from the present analysis.

It is assumed that the orthotropic core undergoes only transverse shear deformations and that a line through the undeformed orthotropic core remains straight under deformations but not necessarily perpendicular to the middle surface of the shell. It is further assumed that the total thickness of the shell element is small compared to its radii of curvature. The face sheets, however, are assumed to satisfy the Kirchhoff-Love assumption and their thicknesses, while not equal, are small compared with the overall thickness of the orthotropic sandwich section. It is likewise assumed that the core compression in a direction normal to the middle surface of the orthotropic sandwich shell is negligible while the properties of each layer are different.

Furthermore, the results for movable edge conditions can be easily derived from the single equation of immovable edge conditions. Numerical results of the critical loads for stabilities of heated sandwich shells have been calculated for movable as well as immovable edge conditions and compared with other available results.

2. Formulation of governing equations

Let us first consider a rectangular Cartesian co-ordinate system x, y, z; x, y in the middle plane of the core, z in the thickness direction (positive downward). If the expression for the strain in the ith face sheet in the x- and y-directions are denoted as ε_{1i} , ε_{2i}, respectively, the transverse shear strain as γ_i, curvature in the x and y directions as κ_1 and κ_2 and the twist as κ_{12}, then the relations (1) hold true for each of the separate face sheets (Fulton, 1961):

$$\begin{aligned}
&\varepsilon_{1i} = u_{ix} + \frac{1}{2}w_x^2 - \frac{w}{R_1},\\
&\varepsilon_{2i} = v_{iy} + \frac{1}{2}w_y^2 - \frac{w}{R_2}\\
&\gamma_i = u_{iy} + v_{ix} + w_x w_y,\ \ i = 1, 2,
\end{aligned} \tag{1}$$