Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
International Journal of Solids and Structures 43 (2006) 4175–4200
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Endochronic theory, non-linear kinematic hardening rule and generalized plasticity: a new interpretation based on generalized normality assumption

Silvano Erlicher [a,*], Nelly Point [a,b]

[a] Laboratoire d'Analyse des Matériaux et Identification LAMI (ENPC/LCPC-Institut Navier) 6 et 8 av. B. Pascal, Cité Descartes, Champs-sur-Marne, 77455 Marne-la-Vallée, Cedex 2, France
[b] Conservatoire National des Arts et Métiers CNAM, Spécialité Mathématiques (442), 292 rue Saint-Martin, 75141 Paris, Cedex 03, France

Received 4 March 2005
Available online 19 April 2005

Abstract

A simple way to define the flow rules of plasticity models is the assumption of generalized normality associated with a suited pseudo-potential function. This approach, however, is not usually employed to formulate endochronic theory and non-linear kinematic (NLK) hardening rules as well as generalized plasticity models. In this paper, generalized normality is used to give a new formulation of these classes of models. As a result, a suited pseudo-potential is introduced for endochronic models and a non-standard description of NLK hardening and generalized plasticity models is also provided. This new formulation allows for an effective investigation of the relationships between these three classes of plasticity models.

Keywords: Thermodynamics of solids; Plasticity; Generalized standard materials; Nonlinear kinematic hardening; Endochronic theory; Generalized plasticity

1. Introduction

The models proposed so far in the literature to describe the rate independent inelastic behavior of real materials subjected to monotonic or cyclic loading conditions can be essentially classified into two main

* Corresponding author. Tel.: +33 1 64 15 37 80; fax: +33 1 64 15 37 41.
E-mail address: erlicher@lami.enpc.fr (S. Erlicher).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2005.03.022

families: (i) models where the present state depends on the present value of observable variables (total strain, temperature) and of suitable internal variables; (ii) models, indicated here as *hereditary*, that require the knowledge of the whole past history of observable variables.

The first group encompasses, for instance, the classical models of Prandtl–Reuss and Prager (see e.g. Lemaitre and Chaboche, 1990) and the NLK hardening model of Armstrong and Frederick (1966), in its original form as well as in the modified versions recently proposed by Chaboche (1991) and Ohno and Wang (1993) in order to improve the ratchetting modelling. For these models, the well known notions of *elastic domain* and *loading* (or *yielding*) *surface* apply. *Associativity* and *non-associativity* of the plastic strain flow rule are also well-established concepts, as well as the assumption of generalized associativity (or *generalized normality*), relating all internal variable flow directions to a given loading surface (Halphen and Nguyen, 1975; Jirásek and Bažant, 2002). Using the language of convex analysis (Rockafellar, 1969), generalized normality entails that the flows of all internal variables belong to the *sub-differential set* of a given scalar non-negative function called *pseudo-potential* (Moreau, 1970; Frémond, 2002).

Among internal variable theories, *generalized plasticity* deserves special attention. A first important step for its formulation was the idea, suggested by Eisenberg and Phillips (1971), of a plasticity model where, despite classical plasticity, loading and yielding surfaces are *not* coincident. Then, starting from an axiomatic approach to describe inelastic behavior of materials, Lubliner proposed some simple generalized plasticity models, able to represent some observed experimental behavior of metals (Lubliner, 1974, 1980, 1984; Lubliner et al., 1993). More recently, generalized plasticity has been used for describing the shape memory alloy behavior (Lubliner and Auricchio, 1996).

Endochronic models (Valanis, 1971) and *Bouc-Wen* type models (Bouc, 1971; Wen, 1976) are two important examples of hereditary models. Endochronic theory has been developed during the seventies and used for modelling the plastic behavior of metals (see, for instance, Valanis, 1971; Valanis and Wu, 1975) and the inelastic behavior of concrete and soils (among others, Bažant and Krizek, 1976; Bažant and Bath, 1976). The endochronic stress evolution rule depends on the so-called *intrinsic time* and is formulated by a convolution integral between the strain tensor and a scalar function of the intrinsic time called *memory kernel*. When the kernel is an exponential function, an incremental form of endochronic flow rules exists, which is commonly used in standard analyses and applications.

Models of Bouc-Wen type are widely employed for modelling the cyclic behavior of structures in seismic engineering (Baber and Wen, 1981; Sivaselvan and Reinhorn, 2000) and for representing hysteresis of magneto-rheological dampers in semi-active control applications (Sain et al., 1997; Jansen and Dyke, 2000). The strict relationship between endochronic and Bouc-Wen type models has been mentioned several times in the literature (see, among others, Karray and Bouc, 1989; Casciati, 1989). Recently, Erlicher and Point (2004) showed that the fundamental element of this relationship is the choice of an appropriate intrinsic time.

Endochronic theory and classical internal variable theory have been compared by using several approaches: Bažant (1978) observed that for endochronic theory the notion of loading surface can still be introduced, but it looses its physical meaning; Valanis (1980) and Watanabe and Atluri (1986) proved that a NLK hardening model can be derived from an endochronic model by imposing a special intrinsic time definition. Moreover, a comparative study between NLK hardening and generalized plasticity models has been presented by Auricchio and Taylor (1995). A tight relationship between endochronic theory and generalized plasticity is also expected to exist, but, by the authors' knowledge, no analysis on this subject has been done. More generally, there is a lack of unified theoretical framework, on which formal comparisons between these plasticity theories could be based. The main goal of this paper is the formulation of this theoretical framework using the classical notion of *generalized normality* (Moreau, 1970; Halphen and Nguyen, 1975). As a result, a new formulation of endochronic and NLK hardening models as well as generalized plasticity models is suggested and is used to investigate the relationships between them.