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Flexural–torsional buckling of misaligned axially moving beams. I. Three-dimensional modeling, equilibria, and bifurcations

Kevin Orloske [a], Michael J. Leamy [b], Robert G. Parker [a,*]

[a] Department of Mechanical Engineering, Ohio State University, 650 Ackerman Road, Columbus, OH 43202, USA
[b] The MITRE Corporation, 7515 Colshire Dr., McLean, VA 22102, USA

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Abstract

The equations of motion for the flexural–flexural–torsional–extensional dynamics of a beam are generalized to the field of axially moving continua by including the effects of translation speed and initial tension. The governing equations are simplified on the basis of physically justifiable assumptions and are shown to reduce to simpler models published in the literature. The resulting nonlinear equations of motion are used to investigate the flexural–torsional buckling of translating continua such as belts and tapes caused by parallel pulley misalignment.

The effect of pulley misalignment on the steady motion (equilibrium) solutions and the bifurcation characteristics of the system are investigated numerically. The system undergoes multiple pitchfork bifurcations as misalignment is increased, with out-of-plane equilibria born at each bifurcation. The amount of misalignment to cause buckling and the post-buckled shapes are determined for various translation speeds and ratios of the flexural stiffnesses in the two bending planes. Increasing translation speed decreases the misalignment necessary to cause flexural–torsional buckling. In Part II of the present work, the stability and vibration characteristics of the planar and non-planar equilibria are analyzed.

Keywords: Axially moving; Beam; Buckling; Stabililty; Flexural-torsional; Bifurcation

1. Introduction

The present work examines the mechanics of translating, beam-like continua that exhibit complex equilibria as a result of boundary misalignment. More specifically, this work considers beams of small aspect ratio for which the bending stiffnesses in two planes have large disparity, examples of which include belt drives, tape drives, and band saw blades. Under the action of boundary misalignment in the plane of larger bending

* Corresponding author. Tel.: +1 614 688 3922; fax: +1 614 292 3163.
E-mail address: parker.242@osu.edu (R.G. Parker).

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stiffness, these axially moving beams experience flexural–torsional buckling into a three-dimensional post-buckled state.

The translating continuum model has direct application to automotive belt-pulley systems undergoing parallel pulley misalignment, where the center of one pulley is displaced in the plane of larger belt bending stiffness. Prediction of a threshold misalignment to cause buckling is necessary to establish tolerances for robust design. Similar boundary misalignment issues arise in tape drives and band saws.

Previous relevant research divides into three areas: flexural–torsional buckling of beams, three-dimensional equilibrium shapes of beams, and flexural–torsional buckling of translating beams. Of these, flexural–torsional buckling of stationary beams occupies the most literature. The present research does not consider thin-walled beams such as I-beams where cross-sectional warping becomes significant (Vlasov, 1961): flexural–torsional buckling of thin-walled beams occupies its own place in the literature (Trahair, 1993) and is not considered.

The first published works on flexural–torsional buckling known to the authors appeared in 1899 (Michell, 1899; Prandtl, 1899) for thin, rectangular, solid beams. Michell considers five different configurations and in each case neglects the bending curvature in the plane of greatest flexural rigidity prior to buckling. Because in each case the beam is loaded parallel to this plane, the effect of bending prior to buckling is neglected. Prandtl develops the same theory as Michell but generalizes it to include the first order effect of the principal bending curvature.

Hodges and Peters (1975) derive a general buckling equation that includes effects not considered by Michell and Prandtl. They then apply a first order approximation to the principal bending curvature and show the resulting buckling equations are actually simpler than those previously published. An historical review of the developments between Michell and Prandtl's work and that of Hodges and Peters is given in Reissner (1979), where transverse shear deformation is included. Milisavljevic (1995) considers the flexural–torsional stability of a cantilever in the presence of shape and load imperfections. Hodges (2001) considers flexural–torsional flutter instabilities that arise from a deep cantilever loaded by a lateral follower force at the tip.

In all works discussed, the flexural–torsional stability of specific systems is analyzed. Michell (1899) considers five systems, Timoshenko (1936) looks at several others, Hodges and Peters (1975) consider only a cantilever with a transverse end load, while Milisavljevic (1995) considers a cantilever with a simultaneous distributed force and an axial force. None include beam translation speed, pre-tensioning, or extension as considered herein. Even without these effects, however, the authors have not found published work on the configuration considered in the present work, namely buckling due to boundary displacement such as pulley misalignment.

Much of the literature on flexural–torsional buckling determines the buckling loads without exploring the three-dimensional shape of the buckled member. Raboud et al. (1996) determined various three-dimensional equilibrium shapes of an inextensible cantilever beam loaded by constant tip or uniform distributed loads. Multiple equilibria are found using a numerical shooting procedure. The potential energies of the post-buckled shapes are used to compare the configurations, but the local stability of each configuration is not addressed. Later, Raboud et al. (2001) examined the stability of the same system except only constant tip loads are considered.

The literature on translating beams is vast, but the literature on flexural–torsional buckling of translating beams is very sparse. The first work in this area was published by Mote (1968), who was motivated by the buckling of band saw blades under edge loads. He models the ends of the beam as simply supported and neglects flexure in the direction of loading. It is shown that transport velocity lowers the critical edge load. After this work little, if any, similar work was done in this area.

Other previous related research falls into the areas of axially moving beams and three-dimensional beam theories. The axially moving beam literature addresses mainly systems undergoing solely transverse or sometimes transverse-extensional motion. A translating beam theory that includes geometric and inertia nonlinearities arising from three-dimensional motion does not exist in the literature. For stationary beams in the absence of initial tension, Crespo da Silva and Glynn (1978) developed such a model for inextensible beams. This work was later generalized to extensible beams (Crespo da Silva, 1988). The three-dimensional translating beam theory developed in the present work further generalizes this model to include translation speed and initial tension. In addition, the present work applies a different reduction scheme than used in Crespo da Silva (1988) to simplify the equations of motion.