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Development mechanism of local plastic buckling in bars subjected to axial impact

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Abstract

In order to clarify the developmental mechanism of the local plastic buckling and the interaction between axial wave and buckling deformation in an axially impacted slender-bar, the non-linear dynamic equations in the incremental form are derived and solved by use of the finite difference method, with the axial wave front treated as a moving boundary. The initial local-buckling deflection given by the characteristic-value analysis is used as the initial condition of the solution of the equations, instead of the initial imperfection that is assumed in literatures. It is found that the initial buckling deflection with one half-wave, occurring near the impacted end, develops into the higher post-buckling mode with several half-waves, as the axial compression waves propagate forward. The numerical results show that no strain reversal occurs at the early stage of post-buckling process, and the solution corresponding to the tangent-modulus theory is valid for the dynamic plastic post-buckling response of the bar at this stage. The theoretical results are in good agreement with the experimental results reported in the literature.

Keywords: Non-linear dynamics; Axial impact; Plastic dynamic buckling; Local buckling development; Twin characteristic parameter

1. Introduction

The plastic buckling problem of straight bars subjected to an axial dynamic loading has been studied extensively (Abrahamson and Goodier, 1966; Hayashi and Sano, 1972a; Lee, 1981; Lindberg and Florence, 1983; Simitses, 1987, 1989; Jones, 1989; Karagiozova and Jones, 1996; Lepik, 2001). For this problem, the earlier analyses usually assumed that the bar is instantaneously brought to the state of uniform compression

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stress (Hayashi and Sano, 1972a; Lindberg and Florence, 1983). Recently, Karagiozova and Jones (1996), and Lepik (2001) investigated, respectively, the influence of stress wave propagation on the elastic-plastic dynamic buckling of the bar under axial impact.

In the analysis for the non-linear dynamic response of the buckling bar, it is usually assumed that the unstressed bar has an initial imperfection, in the shape of a half sine wave, distributed along the entire bar (Hayashi and Sano, 1972a,b). In the calculation of integrating the motion equations, the imperfection is used as the initial condition of the solution of the equations. However, the test results (Hayashi and Sano, 1972b) of axial impact buckling for the slender bar show that, when the impact velocity is high, the local buckling occurs near the impact end at an early stage of the impact process. The same situation is illustrated by a sequence of high speed photographs for the dynamic buckling of an aluminum alloy strip under axial impact (Lindberg and Florence, 1983; see Fig. 2.15 in Lindberg and Florence, 1983).

In Wang and Tian (2002, 2003a,b, 2005), the twin-characteristic-parameter method was presented to clarify the mechanism of buckling initiation and obtain the critical conditions for the geometrically perfect bar as well as the cylindrical shell subjected to an axial dynamic loading. From the analysis by use of the twin-characteristic-parameter method, it is found that, when a geometrically perfect slender-bar is subjected to the axial impact with a high velocity, an initial local-buckling will occur, near the impact end, at the early stage of the process for the axial compression wave to propagate, from the impact end, towards another end of the bar. Assuming that the process of the impact for the bar is initiated at the instant $t = 0$, we define the instant $i = t_{cr}$, at which an infinitesimal buckling deflection occurs near the impact end, as the critical buckling instant. In the duration from $t = 0$ to $t = t_{cr}$, the axial compression wave has traveled the distance L_{cr} in the bar.We define L_{cr} as the critical buckling length. At the critical instant $t = t_{cr}$, the infinitesimal buckling deflection is limited to the region of the length L_{cr} near the impact end, and the part of the bar before the front of the compression wave remains undisturbed. The critical buckling instant t_{cr} and the mode of the infinitesimal buckling deflection can be obtained from the analysis for the critical state by use of twin-characteristic-parameter method (Wang and Tian, 2002, 2003a, in press).

In order to clarify the developmental mechanism of the local buckling deformation in the axially impacted slender-bar, it is quite natural that the initial local-buckling deflection is used as the initial condition of the solution for the dynamic post-buckling response, instead of the initial imperfection distributed along the entire bar. In this way, the elastic dynamic post-buckling response of the slender bar subjected to the axial impact was investigated (Wang and Tian, in press). The theoretical results are in good agreement with the experimental results reported (Lindberg and Florence, 1983; Hayashi and Sano, 1972b) respectively. For the aluminum alloy strip (Lindberg and Florence, 1983), the length of the first half-wave of the post-buckling mode, predicted by the theoretical analysis of Wang and Tian (in press), is equal to 11.2 mm, and is close to the experimental value of 11.9 mm given by Lindberg and Florence (1983). For the Ni–Cr steel bar impacted axially by a striking mass with the velocity $v_0 = 6.3$ m/s, the analysis of Wang and Tian (in press) gives that, at the post-buckling stage, the maximum bending moment appears at the position $x/L = 0.05238$, where L is the length of the bar and x the axial coordinate. In comparison, the experimental result of Hayashi and Sano (1972b) is that $x/L = 0.05$.

In this paper, in order to clarify the developmental mechanism of the local plastic buckling and the interaction between axial wave and buckling deformation in the bar subjected to the axial impact against a rigid wall, the non-linear dynamic equations in the incremental form are derived and solved by use of the finite difference method, with the axial wave front treated as a moving boundary. The initial plastic buckling mode with a small amplitude parameter, obtained by use of the twin-characteristic-parameter solution, is applied as the initial condition of the solution of the non-linear dynamic equations.

In the present analysis, it is assume that the bar is made of the linear strain-hardening material. The tangent-modulus theory and the double-modulus theory are applied, respectively, for describing the relation between the bending-moment and curvature in the dynamic plastic post-buckling deformation. In the relation between the axial stress-increment and strain-increment, both the loading case due to compressive-