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Periodic set of the interface cracks with contact zones in an anisotropic bimaterial subjected to a uniform tension–shear loading

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Abstract

A closed form solution to the plane problem of the theory of elasticity for an infinite anisotropic bimaterial space (plane) with a periodic set of the interface cracks with frictionless contact zones near its tips is obtained. By means of the complex function presentation the problem is reduced to the combined Dirichlet–Riemann boundary value problem for a sectionally holomorphic function and solved exactly. The equations for the determination of the contact zone lengths as well as the closed form expressions for the stress intensity factors are carried out. The variation of the mentioned values with respect to the distance between the cracks is illustrated in table and graphical forms.

Keywords: Periodic interface crack; Anisotropic bimaterial; Contact zone; Stress intensity factors

1. Introduction

In recent years, composite materials and adhesive or bonded joints are being used in wide range of engineering field. In process of such joints manufacture and exploitation numerous defects occur at the material interface. In some cases, these defects are distributed periodically or they can be approximately considered as periodical. As fracture is usually originated from such defects, the problem of the periodic set of the interface cracks is quite important for the applications.

There is numerous literature dealing with the investigation of interface cracks with contact zones near its tips that points to the importance of such investigations for the fracture mechanics of composite materials, because interface cracks are usually the main cause of brittle failure. There are two main models of the interface cracks. First model (oscillatory model) initially assumes that the crack is completely open. Such assumption leads to the oscillatory singularities in the mechanical fields near the crack tips and, consequently, to the complex valued stress intensity factors (SIF). Second model (contact model), proposed by Comninou (1977),

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allows for the contact zones near the crack tips. As was shown by Comninou (1977, 1978) this model leads to the square root singularities in the mechanical fields at the crack tips and the SIFs acquire the real values.

The contact model, which is physically more adequate, has been widely investigated in the literature and some important results were obtained for isotropic bimaterials: see e.g., Simonov (1985), Gautesen and Dundurs (1987), Dundurs and Gautesen (1988), Gautesen (1993). Particularly, it was shown in these papers that the contact zone is usually extremely small for a pure tensile loading. However, appearance of the shear field leads to the decreasing or increasing of the contact zone length depending on the shear loading direction. In some cases the contact zone can attain about 1/3 of the crack length.

The problem of an interface crack between two different anisotropic materials is essentially complicated, and that is why it was not investigated so actively as for isotropic dissimilar materials. An analytical solution of the problem in question in the framework of the classical interface crack model has been obtained by Clements (1971) and afterwards explicitly investigated by Ting (1986), Hwu (1993), Qian and Sun (1998). It has been shown that the assumption of an open crack in most cases leads to the physically unacceptable phenomenon of oscillation.

For anisotropic materials the contact model of an interface crack was considered by Wang and Choi (1983) who reduced the problem to a singular integral equation which was solved numerically. An analytical solution for interface cracks under a combined tension–shear loading was found by Ni and Nemat-Nasser (1991). Herrmann and Loboda (1999) obtained the closed form solution for a crack with a contact zone between orthotropic half-spaces where sufficiently simple transcendental equations for the contact zone length and the formulas for the SIFs were deduced. The problem for several interface cracks with contact zones between dissimilar anisotropic materials was solved by Kharun and Loboda (2004).

The periodic set of the interface cracks between two isotropic materials under "contact" model assumption has been investigated recently by Kozinov et al. (2006). To the authors' knowledge for an anisotropic bimaterial such investigation has not yet been made. In the present paper, an exact analytical solution for the periodic set of the interface cracks with contact zones in an anisotropic bimaterial has been found. Simple equations for the contact zone length determination and the associated SIFs are presented.

2. Statement of the problem

Consider an infinite bimaterial medium containing periodic set of the interface cracks, as shown in Fig. 1. The materials are assumed to be anisotropic with the compliance constants $s_{ij}^{(k)}$, where $k = 1$ is related to the "upper" material and $k = 2$ to the "lower" one.

The medium is subjected to a uniformly distributed tension–shear (σ–τ) loading at infinity. It is known (Comninou, 1977) that interface cracks possess contact zones near tips and these contact zones are, as a rule, extremely small. As was shown by Gautesen (1993), Kharun and Loboda (2004) one of the contact zones has negligibly small influence on the length of another zone and the correspondent SIFs. Therefore, in determining SIFs at some crack tip, one can take into account the contact zone existing near this tip only. This assumption essentially simplifies the mathematical analysis of the problem in question. The following denotation is

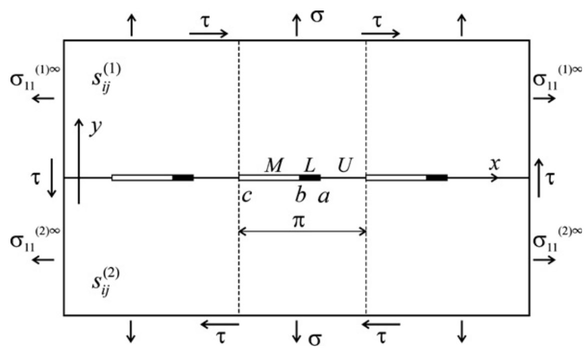

Fig. 1. Geometry of the problem.