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Finite-part integral and boundary element method to solve three-dimensional crack problems in piezoelectric materials

T.Y. Qin [a,*], Y.S. Yu [a], N.A. Noda [b]

[a] College of Science, China Agricultural University, Beijing 100083, PR China
[b] Department of Mechanical Engineering, Kyushu Institute of Technology, Kitakyushu 804-8550, Japan

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Abstract

Using the hypersingular integral equation method based on body force method, a planar crack in a three-dimensional transversely isotropic piezoelectric solid under mechanical and electrical loads is analyzed. This crack problem is reduced to solve a set of hypersingular integral equations. Compare with the crack problems in elastic isotropic materials, it is shown that for the impermeable crack, the intensity factors for piezoelectric materials can be obtained from those for elastic isotropic materials. Based on the exact analytical solution of the singular stresses and electrical displacements near the crack front, the numerical method of the hypersingular integral equation is proposed by the finite-part integral method and boundary element method, which the square root models of the displacement and electric potential discontinuities in elements near the crack front are applied. Finally, the numerical solutions of the stress and electric field intensity factors of some examples are given.

Keywords: Piezoelectric; Crack; Body force method; Boundary element method; Hypersingular integral equation

1. Introduction

The piezoelectric materials have coupled effects between the elastic and the electric fields, and have become of major interest as the functional materials such as actuators and sensors. It is possible to make a system of intelligent composite materials by combining these piezoelectric materials with structural materials. On the other hand, both electrical and mechanical disturbances are present in piezoelectric components, and the strength of the piezoelectric materials is weakened by the presence of defects such as voids and cracks. The reliability of these structures depends on the knowledge of applied mechanical and electric disturbances. When cracks are present, they may grow under service load and affect the performance of structures. Due to the disadvantage of brittleness and low fracture toughness of piezoelectric materials, a considerable number of research works have been carried out to investigate the fracture behavior (Deeg, 1980; Pak, 1990; Suo and

* Corresponding author. Fax: +86 10 62736777.
E-mail address: tyqin@cau.edu.cn (T.Y. Qin).

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Kuo et al., 1992; Wang, 1992; Norris, 1994; Park and Sun, 1995; Shang et al., 2003; Kumar and Singh, 1996; Hill and Farris, 1998; McMeeking, 1999; Qin, 2001; Wang and Huang, 1995; Liu and Fan, 2001; Rajapakse and Xu, 2001; Khutoryansky and Sosa, 1995; Dunn and Wienecke, 1996; Daros and Antes, 2000; Chen and Lin, 1995; Wang and Zhang, 2005).

Because of mathematical difficulties to treat the coupled electromechanical fields in piezoelectricity, the majority of the literature concerning crack problems is based on two-dimensional assumptions. Comparatively, few exact solutions are available in the literature for three-dimensional crack problems in piezoelectric materials. Wang and Huang (1995) obtained the solution for an elliptical crack under uniform tractions and electric disturbance, if the plane of transversal isotropy is parallel to the crack. Closed-form solutions for other 3D crack configurations in an infinite piezoelectric body are yet to be found. Thus, to assess crack-like defects in piezoelectric materials under combined mechanical and electric loadings more efficiently, it is necessary to establish appropriate numerical tools. There are two important numerical methods. One is the finite element method (FEM), and another is the boundary element method (BEM). Shang et al. (2003) have analyzed penny-shaped and elliptical cracks subjected to combined mechanical tension and electric fields by FEM, and presented some numerical results of the stress intensity factors and energy release rates. BEM is a powerful tool for the solution of field problems of mathematical physics, since it offers some inherent advantages over FEM, like the discretization of the boundary only and an improved accuracy in flux calculations. Many publications have already been devoted to the development of fundamental solutions and BEM for piezoelectricity (Deeg, 1980, 2001,), but only a very limited number of them deals with three-dimensional analyses, due to the problems involved resulting from the anisotropy of piezoelectric materials. A 3D Green's function for static piezoelectricity and its derivatives have been presented by Deeg (1980) for piezoelectrics of general anisotropy. Dynamic piezoelectric Green's functions have been presented by Norris (1994) in the frequency domain and by Khutoryansky and Sosa (1995) in the time domain. For the particular case of transversely isotropic piezoelectricity, Dunn and Wienecke (1996) for piezoelectrostatics, and Daros and Antes (2000) for transient analysis developed simplified expressions for the Green's functions. BEM for static piezoelectricity with corresponding numerical results for 3D analysis has been presented by Chen and Lin (1995), and by Hill and Farris (1998). Wang and Zhang (2005) have applied the electrical field saturation model to the fracture prediction of piezoelectric materials containing electrically impermeable cracks, and obtained the stress intensity factor and the energy release rate in closed-form. Zhao and Shen et al. (1997) has investigated the crack problems in piezoelectric materials by BEM and hypersingular integral equation, and given a solution for circular crack. A set hypersingular integral equations and some numerical results for a planar crack in an infinite transversely isotropic piezoelectric media has been given by Chen (2003), in which the unknown function is approximated with a product of the fundamental density function and polynomials. Qin and Noda (2004) have derived a set of hypersingular integral equations of a three-dimensional crack problem in piezoelectric materials, and obtained the exact analytical solutions of the singular stresses and electrical displacements near the crack front in a transversely isotropic piezoelectric solid, but not given the numerical method and solutions.

In this paper, based on the exact analytical solution of the singular stresses and electrical displacements near the crack front, a numerical method for the crack problems in a three-dimensional transversely isotropic piezoelectric solid was proposed by the finite-part integral method and boundary element method. It is shown that for impermeable cracks, the numerical values of the dimensionless intensity factors of K_{I} and K_{IV} are equal to that of the dimensionless intensity factor of mode I for elastic isotropic materials.

2. Basic of piezoelectricity

The linear governing equations and constitutive relations for a piezoelectric material in static equilibrium can be expressed as two separate equations, one representing conservation of momentum and the other conservation of electric charge (Deeg, 1980; Pak, 1990; Suo and Kuo et al., 1992; Wang, 1992). To use these two equations in conjunction with the developed boundary integral equation method, they are combined into one. In these equations, lowercase indices i,l can have values of 1, 2, or 3, and uppercase indices I can take on values of 1, 2, 3, and 4. The modified governing equation for the piezoelectric material in static equilibrium can be written as (Deeg, 1980)