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General framework for the identification of constitutive parameters from full-field measurements in linear elasticity

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Abstract

In this paper, several approaches available in the literature for identifying the constitutive parameters of linear elastic materials from full-field measurements are presented and their sensitivity to a white noise added to the data is compared. The first investigated approach is the virtual fields method (VFM). It is shown that the uncertainty of the parameters identified with the VFM when a white noise is added to the data depends on the choice of a relevant set of virtual fields. Optimal virtual fields exist, thus minimizing the uncertainty and providing the "maximum likelihood solution". The other approaches investigated in this paper are based on finite element model updating (FEMU). It is proved that FEMU approaches actually yield equations similar to the ones derived from the VFM, but with nonoptimal sets of virtual fields. Therefore, the FEMU approaches do not provide the "maximum likelihood solution". However, the uncertainty of FEMU approaches varies dramatically with the cost function to minimize. On one hand, the FEMU approach based on the "displacement gap" minimization yields equations which are very close to the ones of the VFM approach and therefore, its uncertainty is almost the same as the VFM one. On the other hand, it is shown that other approaches based on the "constitutive equation gap" minimization or the "equilibrium gap" minimization provide biased solutions. For all the approaches, very fast algorithms, converging in only two iterations, have been devised. They are finally applied to real experimental data obtained on an orthotropic composite material. Results confirm the success of two methods: the VFM approach which provides the "maximum likelihood solution" and the FEMU approach based on the "displacement gap" minimization.

Keywords: Material identification; Linear elasticity; Finite element model; Virtual fields method; Finite element updating; Uncertainty estimation

1. Introduction

Evaluating accurately and efficiently the parameters which govern the mechanical constitutive equations of materials is essential for the design of structural components. Although standard tests exist for isotropic and macroscopically homogeneous materials, the task is much more complicated for anisotropic and heterogeneous materials. Firstly, the number of parameters to identify is sometimes large enough to make the identi-

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List of notations and symbols

Roman letters

$[A]$	matrix of the system of equation derived by any method for identifying the unknown constitutive parameters
$\{a^e\}$	nodal DoFs of any displacement field in a given finite element
$\{a^{*e}\}$	nodal DoFs of any virtual displacement field in a given finite element
$\{a\}$	nodal DoFs of any displacement field across the whole volume
$\{\tilde{a}\}$	nodal DoFs identified from the measured displacement data
$\{a^*\}$	nodal DoFs of a virtual displacement field
$\{a_p\}$	nodal DoFs of a displacement field computed for a solid whose constitutive parameters are given by vector $\{p\}$
$\{a_{\tilde{p}}\}$	nodal DoFs of a displacement field computed for a solid whose constitutive parameters are given by vector $\{\tilde{p}\}$
$\{a_r\}$	random part of $\{\tilde{a}\}$ divided by σ_u
$\{B\}$	second member of the system of equation derived by any method for identifying the unknown constitutive parameters
$\{b^e\}$	column vector of the nodal forces of a given finite element
$\{b\}$	full assembled column vector of nodal forces It is split in two vectors, one with the known nodal forces (denoted $\{\tilde{F}\}$), and the other, made of the remaining components (not used)
$\mathcal{C}$	fourth order Hooke tensor
$\mathcal{C}^n$	Partial derivative of the fourth order Hooke tensor with regard to a given constitutive parameter p_n
e	finite element
$\{\tilde{F}\}$	part of the decomposition of vector $\{b\}$ where all the unknown nodal forces are picked out
$\{F\}$	exact value for the nodal forces
$\{F_r\}$	random part of $\{\tilde{F}\}$
$\boldsymbol{f}$	field of body forces
$\{G_{p_n}\}$	Partial derivative of $\{U_p\}$ with regard to a given constitutive parameter p_n
$[\mathcal{K}_p^e]$	elementary stiffness matrix of a given finite element built with any vector $\{p\}$ for the constitutive parameters
$[\mathcal{K}_{p_n}^e]$	Partial derivative of $[\mathcal{K}_p^e]$ with regard to a given constitutive parameter p_n
$[\mathcal{K}_p]$	full assembled stiffness matrix of the whole solid with any vector $\{p\}$ for the constitutive parameters
$[\mathcal{K}_{\tilde{p}}]$	full assembled stiffness matrix of the whole solid with any vector $\{p\}$ for the constitutive parameters
$[\mathcal{K}_{p_n}]$	Partial derivative of $[K_p]$ with regard to a given constitutive parameter p_n
$[K]$	first part of the decomposition of the stiffness matrix, referring to the decomposition of vector $\{b\}$
$[K_{p_n}]$	similar as $[K]$
$[K_{\tilde{p}}]$	similar as $[K]$
$[K_{\hat{p}}]$	similar as $[K]$
$[K_{\hat{p}}]$	similar as $[K]$
$[L]$	second part of the decomposition of the stiffness matrix, referring to the decomposition of vector $\{b\}$
$[L_{p_n}]$	similar as $[L]$
$[L_{\tilde{p}}]$	similar as $[L]$
L	length of the region of interest in the shear/bending specimen
$[\widehat{M}^*]$	concatenation of all the vectors of virtual DoFs as a matrix
$[\tilde{M}]$	concatenation of all vectors $\{[K_{p_n}]\{\tilde{U}\} + [L_{p_n}]\{\tilde{V}\}\}$ as a matrix