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The stress intensities of three-dimensional corner singularities in a laminated composite

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Abstract

The stress intensity of three-dimensional corner singularity is computed for the tip of a transverse crack terminating on the free surface in a laminated composite. Firstly, stress singularity is calculated via finite element method applied for the angular domain. Then the two-state M-integral is employed, in conjunction with eigenfunction expansion, for computing the stress intensity of the stress singularity. The numerical example demonstrates the effectiveness of the proposed computational scheme.

Keywords: Three-dimensional corner singularity; Transverse crack; Laminated composite; Stress intensity; Conservation integrals

1. Introduction

The determination of stress intensities as well as the stress singularities of singular stress fields around generic wedges in linear elastic materials has been a major subject in fracture mechanics. In three-dimensional wedges, however, most works did not look into the near-tip stress intensities of the singular stress field, but concentrated on calculating only the stress singularities. This is partly because three-dimensional problems are themselves very complicated and partly because any reliable methodology to compute stress intensities or a fracture parameter like the J-integral for three-dimensional cracks was not available.

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There are many researchers that have discussed the order of stress singularities on three-dimensional wedge vertices. For example, Koguchi and Muramoto (2000), Picu and Gupta (1997), Ghahremani (1991), Somaratna and Ting (1986), Bazant and Estenssoro (1979), Benthem (1980, 1977) and others cited in these papers. Only a few studies have tried to compute the stress intensities as well as the stress singularities. For example, near the vertex of a thin plate with a crack, Nakamura and Parks (1989, 1988) introduced the corner stress intensity which is computed from the three-dimensional local J-integral. However, this method is applicable only for three-dimensional cracks. Labossiere and Dunn (2001) conducted a series of very elaborate testing to confirm that the near-tip stress intensities of the singular fields on the bimaterial free edges are accurately correlated to the initiation of fractures in the specimens. Furthermore, they showed that the intensities of the singular stresses around the three-dimensional wedge on the interface corner of the two joining materials are in an excellent correlation with the initiation of the failure at the wedge vertices of the specimens. Hence, of paramount importance is an efficient and accurate calculation of these near-tip intensities. Recently Lee and Im (2003) proposed a systematic computational scheme, which is a method to calculate the near-tip intensities of the singular stress fields around the three-dimensional wedges with the aid of the two-state M-integral.

The computation of the near-tip intensities of the singular fields may be rather straightforward for the two-dimensional wedges, and there are many schemes available (see Im and Kim, 2000 and the papers cited therein, for examples). Among others, the application of the two-state conservation integral (Im and Kim, 2000; Kim et al., 2001; Jeon and Im, 2001; Lee et al., 2001) is known to be a robust method of computation. Moreover, Lee and Im (2003) showed that the computational scheme using the two-state M-integral is applicable for three-dimensional wedges by computing the near-tip intensities for the vertex of a thick plate with a crack, and the bimaterial interface corner, which was considered by Labossiere and Dunn (2001).

The purpose of the present paper is to extend the computation of the near-tip intensities around three-dimensional wedges reported in our previous work (Lee and Im, 2003) to the case of anisotropic materials. A particular emphasis is given to the extension of the approach to the case of the three-dimensional crack corner of a laminated composite with transverse cracks or on the intersection of transverse cracks with free surface in composite laminates, which was treated by Somaratna and Ting (1986) and Ghahremani (1991). A brief review is first stated for the eigenfunction expansion of the solution for three-dimensional elastic wedges. This is followed by a summary regarding the two-state M-integral. The two-state M-integral is then applied for calculating the near-tip intensity by utilizing the complementarity relationship for the eigenvalues of the three-dimensional wedges. That is, the path or surface independence property of the two-state M-integral for a pair of the complementary eigenvalues is exploited to equate its value calculated on the vanishing near-field surface around the vertex to the value from finite element analysis on the far-field surface. This procedure enables us to calculate the intensity of the singular stress field around the wedge vertex in an efficient manner.

For the numerical example, we choose a crack corner of a laminated composite with transverse cracks, of which the stress singularities were discussed by Ghahremani (1991), and Somaratna and Ting (1986). This example demonstrates the effectiveness and accuracy of the proposed-scheme.

2. Eigenfunction expansion of the solution for three-dimensional elastic wedges

Let the stress and strain components in the spherical coordinates be represented by 1-D arrays as follows:

$$(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6) = (\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\phi\phi}, \sigma_{\theta\phi}, \sigma_{r\phi}, \sigma_{r\theta}) \tag{1a}$$

$$(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5, \varepsilon_6) = (\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{\phi\phi}, 2\varepsilon_{\theta\phi}, 2\varepsilon_{r\phi}, 2\varepsilon_{r\theta}) \tag{1b}$$