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International Journal of Solids and Structures 43 (2006) 6751–6766

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

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Solution of the moiré hole drilling method using a finite-element-method-based approach

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Received 1 September 2005; received in revised form 4 February 2006
Available online 6 March 2006

Communicated by Stelios Kyriakides

Abstract

The moiré hole drilling method in a biaxially loaded infinite plate in plane stress is an inverse problem that exhibits a dual nature: the first problem results from first drilling the circular hole and then applying the biaxial loads, while the other problem arises from doing the opposite, i.e., first applying the biaxial load and then drilling the circular hole. The first problem is hardly ever addressed in the literature but implies that either separation of stresses or material property identification may be achieved from interpreting the moiré signature around the hole. The second is the well-known problem of determination of residual stresses from interpreting the moiré fringe orders around the hole. This paper addresses these inverse problem solutions using the finite element method as the means to model the plate with a hole, rather than the typical approach using the Kirsch solution, and a least-squares optimization approach to resolve for the quantities of interest. To test the viability of the proposed method three numerical simulations and one experimental result in a finite width plate are used to illustrate the techniques. The results are found to be in excellent agreement. The simulations employ noisy data to test the robustness of this approach. The finite-element-method-based inverse problem approach employed in this paper has the potential for use in applications where the specimen shape and boundary conditions do not conform to symmetric or well-used shapes. Also, it is a first step in testing similar procedures in three-dimensional samples to assess the residual stresses in materials.

Keywords: Inverse problem; Finite element analysis; Moiré; Hole drilling; Residual stresses; Optimization; Least squares approach

1. Introduction

The use of the hole drilling method is well known to practitioners as a means to determine residual stresses using a strain gage rosette (ASTM, 1998). Additionally, various researchers have studied obtaining

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doi:10.1016/j.ijsolstr.2006.02.010

residual stresses by moiré hole drilling (Cárdenas-García and Verhaegh, 1999; Cárdenas-García and Verhaegh, 1999; Furgiuele et al., 1991; Nicoletto, 1988; Nicoletto, 1991; Schwarz et al., 2000; Wu et al., 1998; Wu et al., 1998), even fully detailing the use of moiré hole drilling for the most general case possible (Cárdenas-García, 2001). All of these solutions rely on the analytical solution by Kirsch (1998) for a homogeneous, isotropic infinite plate with a hole in plane stress (Kirsch, 1998). This presents different practical limitations, such as satisfying the theoretical requirement that the size of the hole needs to be small enough relative to the size of the specimen so as to approach the required infinite plate size. This paper has several objectives in revisiting the moiré hole drilling method: (a) to introduce the finite element method as a means to model the plate material so as to preclude any size or shape limitations on the model; (b) to show that an overdeterministic approach may be used to combine experimental data with the finite element method to solve the residual stress problem; and (c) to expand the perspective of the moiré hole drilling method so as to include not only residual stress determination but also stress separation and determination of material properties.

2. The hole method

Initially we would like to look at the hole drilling problem from a more general perspective by using the concepts of direct or forward problem, and inverse or backward problem. A direct or forward problem refers to the determination of outputs or responses for a well-defined system using knowledge of the inputs or sources. This is typical of a solution such as the one elaborated by Kirsch (1998) mentioned above in relation to an infinite plate with hole, i.e., given inputs: boundary conditions and elastic material properties, that relate to a specified geometry such as that of an infinite plate with a hole, yields outputs: the displacements and/or strains around the hole. Fig. 1 illustrates this with a schematic portraying the application of the superposition principle in the case of general loading of an infinite plate with a hole. Practically this means that the hole is drilled first and then the loads are applied. We refer to this situation as the "pre-existing hole" problem. Note that the applied far field stresses σ_X, σ_Y, and τ_{XY} may be replaced by the principal stresses σ_1 and σ_2 which are normal to each other and are oriented at an angle ϕ from the horizontal direction. This equivalent loading condition is obtained using the Mohr's circle equations.

This Kirsch (1998) solution example allows us to address how an inverse problem approach is related to it. An inverse or backward problem approach is concerned with the determination of inputs or sources for a well-defined system from observed outputs or responses. We are able to define two different inverse problems in reference to this forward problem. The first inverse problem requires observed outputs: displacements or strains and knowledge of elastic material properties to obtain the boundary conditions in the form of applied biaxial stresses. This inverse problem approach is a means to separate the principal stresses by means of a "pre-existing hole". The second inverse problem requires that observed outputs: displacements or strains and knowledge of the boundary conditions as applied biaxial loads are required to obtain the elastic material properties. An advantage of this approach is the simultaneous determination of the elastic properties from the moiré signature around the hole in a biaxially loaded plate (Cárdenas-García, 2000, 2001) at one general biaxial load level. This last approach is a non-traditional application of the hole method that finds practical

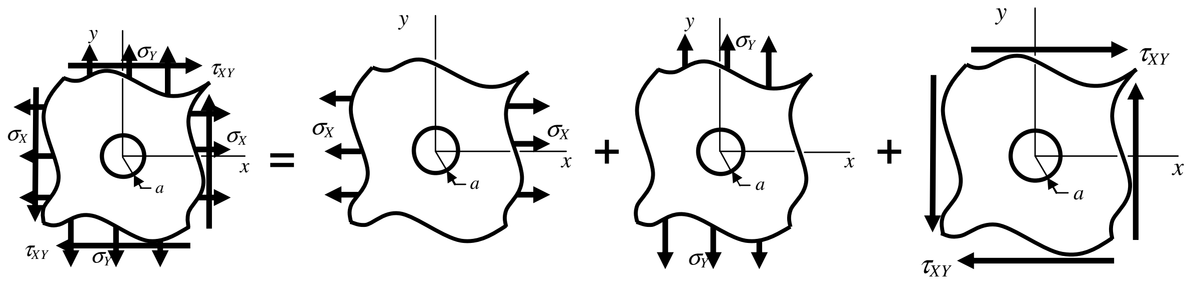

Fig. 1. Application of the superposition principle for a generally loaded infinite plate with a hole in plane stress.