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A semi-infinite chamfered contact solution and its application to almost complete contacts

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Abstract

The solution for a semi-infinite rigid block having a flat face but with a small, shallow edge chamfer, and pressed onto an incompressible half-plane, is considered. The surface traction distribution and internal state of stress under both normal and a monotonically increasing shearing force are found, and the characteristics of the solution explored. As an example it is employed to find the edge-solution for a finite square-ended but chamfered punch in contact with a half-plane.

Keywords: Asymptotics; Logarithmic singularity; Partial slip; Chamfered contact; Contact mechanics

1. Introduction

Semi-infinite contact asymptotes are valuable solutions in the study of contacts generally and fretting-type problems in particular. Contacts can be classified as 'incomplete' or 'complete'. The contact pressure at the edge of an incomplete contact goes continuously to zero at the contact edge and there is always a region of partial slip when an oscillatory shear force is present. The contact is complete when the surfaces conform and a discontinuity in profile gradient defines the contact edge. In the latter case there is always an implied elastic power-order singularity in the contact pressure, and the contact will normally shake down (in a frictional sense) to a state of complete adhesion (Churchman and Hills, 2005). In practice, there will always be *some* edge radius or chamfer, and this will have the effect of making the contact pressure very locally bounded. Asymptotic solutions can play a big rôle in refining the solution here: they can be used both to add detail to what is nominally a complete contact but with an edge detail of the kind described, and they can be used to classify the region in which a crack may nucleate as either incomplete or complete, depending on the size of the plastic zone and the extent of slip.

Although the solution for the contact pressure at the edge of a chamfered contact has several similarities with the 'semi-infinite flat and rounded punch solution' (Dini and Hills, 2004; Dini et al., 2005), it has at least

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one major difference, viz. that, because the slope of the punch face is discontinuous, a logarithmic singularity in the contact pressure arises quite close to the contact edge, so that the rate at which the bounded behaviour decays away from the contact edge is rather steeper. This will emerge as part of the solution, together with its general characteristics. As an example the solution is applied to a slightly chamfered rigid punch.

Earlier solutions to bimaterial problems involving logarithmic singularities were found by Dempsey and Sinclair (1981) and the subject of the "lapping round" of material onto an inclined contact face has been analysed by Adams (1979).

1.1. Formulation

The problem to be solved is depicted in the "zoomed in" region in Fig. 1. An elastic half-plane is indented by a semi-infinite punch having a flat face, save for a small portion near the free edge: this has an exterior angle ψ. There are several ways in which the contact problem may be solved, but here we simply take the solution for contact between an elastic wedge of exterior angle ψ, tilted at an angle α to the half-plane normal (Sackfield et al., 2005), and take the limit $\alpha \to \psi/2$, bringing one flank into glancing contact, giving

$$p(x) = K_L \ln \left| \frac{\sqrt{1 + x/b} - 1}{\sqrt{1 + x/b} + 1} \right|, \quad \text{for} \quad -b > x > \infty, \tag{1}$$

where

$$K_L = \frac{\psi}{\pi A}, \tag{2}$$

b is the length of contact in the chamfered region, and A is the composite compliance of the two bodies.

The original paper used uncoupled half-plane theory for each contacting body, and the solution is therefore appropriate when both (a) $\psi \lll \pi$, and (b) either no interfacial shearing tractions arise or because Dundurs' second constant for the material pair vanishes. The last requirement is most usually fulfilled when either the components are made from the same material or one is rigid and the other incompressible. However, as will be

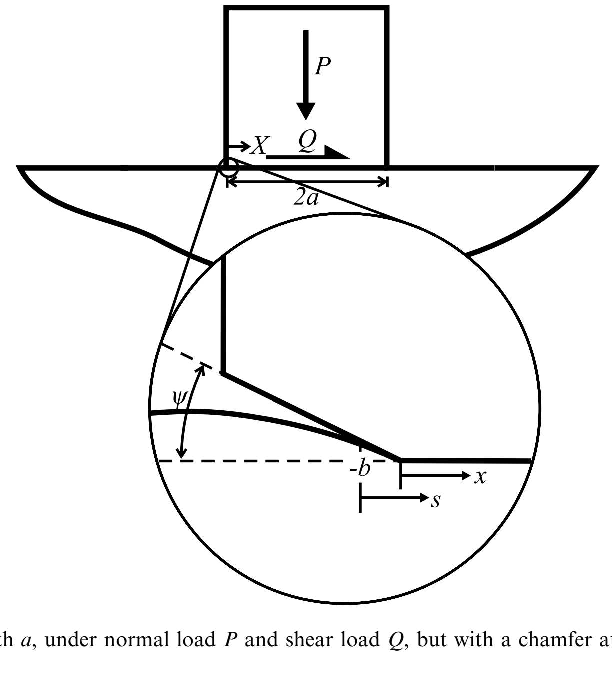

Fig. 1. Rigid punch of half-width a, under normal load P and shear load Q, but with a chamfer at the extreme contact edge (inset).