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Coplanar propagation paths of 3D cracks in infinite bodies loaded in shear

Elie Favier, Véronique Lazarus *, Jean-Baptiste Leblond

Laboratoire de Modélisation en Mécanique, Université Pierre et Marie Curie (Paris VI), Tour 65-55, Case 162, 4 place Jussieu, 75252 Paris Cedex 05, France

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Abstract

Bower and Ortiz, recently followed by Lazarus, developed a powerful method, based on a theoretical work of Rice, for numerical simulation of planar propagation paths of mode 1 cracks in infinite isotropic elastic bodies. The efficiency of this method arose from the need for the sole 1D meshing of the crack front. This paper presents an extension of Rice's theoretical work and the associated numerical scheme to mixed-mode (2 + 3) shear loadings. Propagation is supposed to be channeled along some weak planar layer and to remain therefore coplanar, as in the case of a geological fault for instance. The capabilities of the method are illustrated by computing the propagation paths of cracks with various initial contours (circular, elliptic, rectangular, heart-shaped) in both fatigue and brittle fracture. The crack quickly reaches a stable, almost elliptic shape in all cases. An approximate but accurate analytic formula for the ratio of the axes of this stable shape is derived.

Keywords: 3D fracture mechanics; Coplanar crack propagation paths; Mode 2 + 3; Brittle fracture; Fatigue; Stable shape; Perturbation method; Weight functions

1. Introduction

There are several more or less efficient methods for numerical simulation of propagation paths of 3D cracks in elastic media. The most classical and general one is the finite element method (FEM). A

* Corresponding author. Tel.: +33 144278717; fax: +33 144275259.
E-mail addresses: favier@lmm.jussieu.fr (E. Favier), vlazarus@ccr.jussieu.fr (V. Lazarus), leblond@lmm.jussieu.fr (J.-B. Leblond).

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non-trivial example involving a complex, non-planar crack shape is provided in the work of Xu et al. (1994). But the FEM requires meshing the whole 3D cracked body at each step of the crack propagation, and it is difficult to do this automatically. A recent and efficient variant of the FEM consists of coupling the level set method and the extended finite element method (XFEM); see for instance, Sukumar et al. (2003), Moes et al. (2002), Gravouil et al. (2002).

The sole meshing of the initial geometry is then required. Another alternative consists of using the boundary element method (BEM), which requires only 2D meshing of the crack surface and the outer boundary. Several examples of such an approach are provided in Chapter 5 of Bonnet (1994)'s book. Methods based on integral equations are especially attractive for infinite bodies since the sole meshing of the crack surface is then necessary; see for instance the examples provided by Fares (1989) and Xu and Ortiz (1993). If, in addition, the crack propagates along a plane, compelling methods requiring the sole 1D meshing of the crack front are envisageable. Using an earlier theoretical work of Rice (1989) and Bower and Ortiz (1990, 1991, 1993) proposed such an approach and applied it to various problems of practical interest. Lazarus (2003) later defined a simplified variant of this method which resulted in no significant loss of numerical accuracy.

More specifically, the basis of Bower and Ortiz' method and Lazarus' variant was Bueckner–Rice's weight function theory. The 2D version of this theory was expounded by Bueckner (1970) and Rice (1972), and its extension to the 3D case by Rice (1972) (in the appendix of this reference) and Bueckner (1973). It was applied by Rice (1989) to planar cracks with arbitrary contours loaded in mode 1. The theory yielded, to first order, the variation of the mode 1 stress intensity factor (SIF) along the crack front arising from some small but otherwise arbitrary coplanar perturbation of this front, under conditions of constant prescribed loading. This variation was expressed as a line integral over the unperturbed front which involved, in addition to the perturbation, some geometry-dependent "kernel" linked to the mode 1 weight function. The theory also provided the variation of this kernel, again to first order, in a similar form. Bower and Ortiz (1990, 1991, 1993)'s method consisted of applying Rice's two formulae to some sequence of small perturbations of the front resulting in arbitrary deformation of its initial shape. They applied it to the study of the propagation paths of semi-infinite tensile cracks in heterogeneous media, in both fatigue and brittle fracture.

In Lazarus (2003)'s variant, the numerical procedure was greatly simplified by using linear instead of quadratic elements, calculating the crack advance and the various integrals at the same discretization points instead of distinguishing between nodes and collocation points, etc. Validation tests based on numerical calculation of SIFs for crack geometries for which some analytical solution existed showed that such simplifications did not affect the overall numerical accuracy. In fact, the only point that was found to really require special care was accurate evaluation of some integrals in principal value. Also, Lazarus replaced the Irwin–Griffith propagation law for brittle fracture used by Bower and Ortiz by that of Paris. Nothing was lost by doing so since as noted by Lazarus, in addition to being a good propagation law for fatigue, Paris' law "simulates" Irwin–Griffith's law in the limit of very large Paris exponent. The advantage of using Paris' law was that the crack advance at all time steps and discretization points was directly provided by some explicit formula. In contrast, deducing the crack advance at the discretization points from the Irwin–Griffith criterion, as was done by Bower and Ortiz, was more difficult in two respects. First, iterations were required to determine the "active" part of the front, that is that portion which effectively propagates at the instant considered. Second, for each iteration, it was necessary to solve a large system of linear equations on the unknown values of the crack front advance at the discretization points of this active part. Lazarus applied her method to the study of the propagation paths, in fatigue and brittle fracture, of initially elliptic, rectangular and heart-shaped cracks loaded through some uniform remote tensile stress. She found that for all of these initial configurations, the crack quickly reached a stable, circular shape.