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Crack spacing effect for a piezoelectric cylinder under electro-mechanical loading or transient heating

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Abstract

This paper investigates the fracture problem of a piezoelectric cylinder with a periodic array of embedded circular cracks. An electro-mechanical fracture mechanics model is established first. The model is further used to the thermal fracture analysis of a piezoelectric cylinder subjected to a sudden heating on its outer surface. The temperature field and the associated thermal stresses and electric displacements are obtained and are added to the crack surface to form a mixed-mode boundary value problem for the electro-mechanical coupling fracture. The stress and stress intensities are investigated for the effect of crack spacing. Strength evaluation of piezoelectric materials under the transient thermal environment is made and thermal shock resistance of the medium is given.

Keywords: Thermal stresses; Fracture mechanics; Piezoelectric materials

1. Introduction

Since piezoelectric materials are very brittle and susceptible to fracture, it is necessary to understand the fracture behaviors of these advanced materials. On the other hand, mechanical, electrical, and thermal fields are coupled in most physical problems. Thermal effects in piezoelectric materials could be important when those materials are used at high or low temperature environments. In a series of their work, Herrmann and Loboda investigated interface cracks with a frictionless contact zone at the crack tip between two semi-infinite piezoelectric spaces under the action of a remote electro-mechanical loading and a temperature flux (Herrmann and Loboda, 2003a,b). Gao et al. (2002) presented an exact solution

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for the problem of an elliptic hole or a crack in a thermopiezoelectric solid. The method used in that paper was also applied to an elliptical cavity in a magnetoelectroelastic solid under remotely uniform in-plane electromagnetic and/or anti-plane mechanical loading (Gao et al., 2004). The thermoelastic behavior of piezoelectric solids with defects, such as holes and interface cracks, were considered by Zhang et al. (2002), and by Qin and Mai (1999). The problem of a 2-D piezoelectric material with an elliptic cavity under uniform heat flow was investigated by Lu et al. (1998). Studied by Niraula and Noda (2002) was a transversely isotropic thermopiezoelastic material strip containing an edge crack under thermal and electrical loading conditions. Recently, the thermally induced fracture problem for a piezoelectric laminate with a crack subject to uniform electric and temperature fields was considered by Ueda (2003). A surface crack in a piezoelectric strip under transient thermal load was investigated by Wang and Mai (2003). The exact solution for a penny-shaped crack in a piezoelectric medium under steady thermal flux was given by Wang and Noda (2004). Shang et al. (2003) investigated propagation behavior of an elliptical crack in thermopiezoelectric material subjected to a uniform temperature. The three-dimensional strain energy density formulation was used to determine the direction of crack propagation and the shape of the initial fracture increment. Gu and Yu (2003) considered the anti-plane problem of thermal effect near crack tip region of piezoelectric material subjected to electrical impact loading by means of the integral transforms and the singular integral equations.

Although a variety of challenging issues related to certain thermal crack problems in the piezoelectric materials have been addressed, one of the remaining problems that need to be fully understood is that of a periodic array of parallel cracks in such media subjected to thermal loading. Past experience suggests that cracks in a medium may be either a single dominant crack or a roughly regular array of periodic cracks (Grot and Martyn, 1981; Rizk, 2004; Timm et al., 2003; Erdogan and Ozturk, 1995; Ishihara and Noda, 2001). Therefore, it is important to consider multiple cracking of piezoelectric media.

This paper investigates the thermal fracture of a piezoelectric cylinder with a periodic array of embedded circular cracks. Both electro-mechanical loads and thermal loads are considered. The crack problem is solved by means of integral equation technique. Effect of crack spacing on the stress and crack front field intensity factors are investigated. The thermal shock (transient thermal loading) resistance of a piezoelectric material is studied for a cylinder specimen subjected to a sudden heating on its outer surface. Some conclusions are drawn.

2. Electro-mechanical model for a row of infinitesimal periodic cracks

This section develops an electro-mechanical model for a periodic array of cracks in a piezoelectric cylinder (Fig. 1). The analytical model is generalized for any distribution of the electro-mechanical loads on the crack faces. The cylindrical coordinates r, θ and z are coincident with the principal axes of the material symmetry. We investigate an axis-symmetric problem such that all the field variables are functions of the radial coordinate r and the axial coordinate z only. Hereafter θ denotes the circumferential coordinate; the symbols and D denote the stress and electric displacement, respectively; u and w are, respectively, the radial and axial components of the displacement vector; and ϕ is the electric potential. Constitutive equations for piezoelectric materials whose poling direction is coincident with the positive z-axis are (Lin et al., 2003):

$$\sigma_{rr} = c_{11}\frac{\partial u}{\partial r} + c_{12}\frac{u}{r} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} - \lambda_{11}T, \tag{1a}$$

$$\sigma_{\theta\theta} = c_{12}\frac{\partial u}{\partial r} + c_{11}\frac{u}{r} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} - \lambda_{11}T, \tag{1b}$$

$$\sigma_{zz} = c_{13}\frac{\partial u}{\partial r} + c_{13}\frac{u}{r} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \phi}{\partial z} - \lambda_{33}T, \tag{1c}$$