



ELSEVIER

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SCIENCE @ DIRECT®

International Journal of Solids and Structures 43 (2006) 2146–2159

INTERNATIONAL JOURNAL OF  
**SOLIDS and  
STRUCTURES**

[www.elsevier.com/locate/ijsolstr](http://www.elsevier.com/locate/ijsolstr)

# Effect of the nonlinear pre-buckling state on the bifurcation point of conical shells

Mahmood Jabareen, Izhak Sheinman \*

*Faculty of Civil and Environmental Engineering, Technion—Israel Institute of Technology, Haifa 32000, Israel*

Received 16 December 2004; received in revised form 22 May 2005

Available online 20 July 2005

## Abstract

The effect of pre-buckling nonlinearity on the bifurcation point of a conical shell is examined on the basis of three shell theories: Donnell's, Sanders' and Timoshenko's. The eigenvalue problem is solved iteratively about the nonlinear equilibrium state up to the bifurcation point. A new algorithm is presented for the real buckling behavior, dispensing with the need to cover the entire nonlinear pattern. This algorithm is very important for structures characterized by a softening process, in which the pre-buckling nonlinearity depresses the buckling level relative to the classical one.

The procedure involves nonlinear partial differential equations, which are separated into two sets (using the perturbation technique) for the pre-buckling and buckling states, respectively and solved with the variable expanded in Fourier series in the circumferential direction, and by finite differences in the axial direction. A general computer code was developed and used in studying the effect of the pre-buckling nonlinearity on the buckling level, of the shell under axial compression, in the context of the three shell theories.

© 2005 Elsevier Ltd. All rights reserved.

**Keywords:** Conical shell; Bifurcation point; Buckling load; Nonlinear analysis; Shell theories

## 1. Introduction

Loss of stability by buckling in shell-like structures is one of the most important and crucial failure phenomena. Treatment of the buckling process as a linear one in this context has been questioned due to the discrepancies observed between theoretical predictions and experimental results. The first reason for these discrepancies is the fact that the structures in question are subjected to considerable nonlinear pre-buckling deformation; consequently, the linear approach is unsuitable for predicting their stability. The second is the

\* Corresponding author. Tel.: +972 4829 3042; fax: +972 4829 5697.

E-mail address: [cvrnrsh@techunix.technion.ac.il](mailto:cvrnrsh@techunix.technion.ac.il) (I. Sheinman).

fact that the load capacity of such structures is strongly affected by the initial imperfection pattern. Accordingly, two main approaches are available for our purpose: (I) consideration of the sensitivity to imperfection, see e.g., Goldfeld et al. (2003); (II) determination of the real buckling load while dispensing with the need to cover the entire nonlinear equilibrium path.

The present paper is concerned with the second approach. For this purpose the conical shell is chosen as a representative structure for the entire range of degree of nonlinearity. Namely, cylindrical shell (conical with vertex half-angle ( $\alpha = 0^\circ$ )) characterized by high nonlinearity, and annular plate ( $\alpha = 90^\circ$ ) characterized by very low nonlinearity. By varying the cone vertex half-angle ( $\alpha$ ) conclusions can be drawn for the entire range of nonlinearity level.

While the linear stability (with the nonlinear pre-buckling deformation disregarded) of cylindrical and conical shells was extensively studied, its nonlinear counterpart has so far, to the best of the authors' knowledge, attracted less interest. Seide (1956) was the first to derive the critical buckling load for an axisymmetric mode in a conical shell. Singer (1965) also used the asymmetric buckling mode and obtained the same buckling load as Seide. The effect of the four possible in-plane boundary conditions on the buckling behavior of a conical shell under axial compression was studied by Singer (1962) and by Baruch et al. (1970). Pariatmono and Chryssanthopoulos (1995) and Spagnoli (2003) showed that at a certain aspect ratio of a conical shell, different buckling modes correspond to the same value of critical buckling. Tong (1994) suggested a simple formula for the critical buckling loads of laminated conical shells, based on Seide's (1956) and assuming constant stiffness. Recently, Goldfeld and Arbocz (2004) studied the influence of variation of the stiffness coefficients on the buckling behavior of laminated conical shells.

So far most of the relevant studies have been limited to the simplified theoretical treatment assuming membrane-like or linear pre-buckling. There are, however, a few studies where the influence of the pre-buckling state is taken into account. Brush (1980) considered the effect of pre-buckling rotation in a cylindrical shell and improved accordingly the buckling load obtained without it. Famili (1965) studied the asymmetric behavior of truncated and complete conical shells under uniform hydrostatic pressure, also taking into account the large deformation in the pre-buckling state. Zhang (1993) studied, in the same manner, the buckling and initial post-buckling behavior, with considering the nonlinear pre-buckling behavior, under axial compression and hydrostatic pressure. The limitation in Famili's and Zhang's works is the assumption of an axisymmetric pre-buckling solution and recourse to  $WF$  formulation (where  $W$  is the normal displacement and  $F$  is the Airy stress function), which may yield inaccurate results as was shown later in Sheinman and Goldfeld (2001). For general structures, using the finite-element technique, Brendel and Ramm (1980) devised an improved scheme for prediction of the critical load, with additional linear buckling analyses carried out at a number of intermediate load levels preceding instability. Chang and Chen (1986) proposed a scheme for prediction the real buckling load based on combination of the linear and nonlinear analyses, and with disregarding the stiffness terms which are quadratically dependent on the generalized displacement.

While extensive literature is devoted to processes characterized by a limit point, very few works cover the entire nonlinearity range. From the analytical point of view, two approaches are used in investigating the effect of pre-buckling nonlinearity: (I) the full nonlinear analysis which yields an exact prediction of the nonlinear bifurcation point by finding the intersection of two (or more) equilibrium paths; this approach entails a heavy computational effort; (II) the eigenvalue approach, focusing on the nonlinear equilibrium state. The eigenvalue problem is solved iteratively until the eigenvalue for the current load equals unity, thus yielding the stiffness and geometric matrices right at the bifurcation point; this approach is substantially more effective and computationally cheaper.

The primary objective of the present paper is a new algorithm for the real buckling load dispensing with the need to cover the entire nonlinear behavior. Due to the significant differences between the various shell theories as regard the classical buckling load (Sheinman and Goldfeld, 2001), the sensitivity to imperfection (Sheinman and Goldfeld, 2003) and the nonlinear behavior till the limit point (Simites et al., 1985),

Download English Version:

<https://daneshyari.com/en/article/280826>

Download Persian Version:

<https://daneshyari.com/article/280826>

[Daneshyari.com](https://daneshyari.com)