Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Solids and Structures 43 (2006) 3832–3855

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Eigenanalysis and continuum modelling of pre-twisted repetitive beam-like structures

N.G. Stephen *, Y. Zhang

School of Engineering Sciences, Mechanical Engineering, University of Southampton, Highfield, Southampton SO17 1BJ, UK

Received 19 May 2005
Available online 11 July 2005

Abstract

A repetitive pin-jointed, pre-twisted structure is analysed using a state variable transfer matrix technique. Within a global coordinate system the transfer matrix is periodic, but introduction of a local coordinate system rotating with nodal cross-sections results in an autonomous transfer matrix for this Floquet system. Eigenanalysis reveals four real unity eigenvalues, indicating tension–torsion coupling, and equivalent continuum properties such as Poisson's ratio, cross-sectional area, torsion constant and the tension–torsion coupling coefficient are determined. A variety of real and complex near diagonal Jordan decompositions are possible for the multiple (eight) complex unity eigenvalues and these are discussed in some detail. Analysis of the associated principal vectors shows that a bending moment produces curvature in the plane of the moment, together with shear deformation in the perpendicular plane, but no bending–bending coupling; the choice of a structure having an equilateral triangular cross-section is thought responsible for this unexpected behaviour, as the equivalent continuum second moments of area are equal about all cross-sectional axes. In addition, an asymmetric stiffness matrix is obtained for bending moment and shearing force coupling, and possible causes are discussed.

Keywords: Pre-twist; Repetitive; Floquet; Jordan canonical forms; Eigenproblem; Tension–torsion; Bending–shear; Continuum; Homogenisation; Properties

1. Introduction

Repetitive (or periodic) structures are analysed most efficiently when that periodicity is taken into account. It is possible to determine the behaviour of the complete structure from analysis of a single repeating

* Corresponding author. Tel.: +44 (0) 23 80592359; fax: +44 (0) 23 80593230.
E-mail address: ngs@soton.ac.uk (N.G. Stephen).

0020-7683/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2005.05.023

Nomenclature

A	cross-sectional area
d	member diameter
$\mathbf{d}$	nodal displacement vector
E	Young's modulus
$\mathbf{F}$	force vector
G, $\mathbf{G}$	shear modulus, transfer matrix
H	height of cell cross-section ($H = \sqrt{3}L/2$)
i, I, $\mathbf{I}$	$\sqrt{-1}$, second moment of area, identity matrix
J, $\mathbf{J}$, $\mathbf{J}_\mathrm{m}$	torsion constant, Jordan block and canonical form, metric
$\mathbf{K}$, K	stiffness matrix, coupling coefficient
L	length of cell, and of cross-sectional members, left
M	bending or twisting moment
n, N, $\mathbf{N}$	index of cell or section, compliance matrix
p	period
Q	shearing force
R	radius of bending curvature, right
$\mathbf{s}$	state vector
T	tensile force
$\mathbf{T}$	orthogonal coordinate transformation matrix
u, v, w	displacements in the x-, y- and z-directions
$\mathbf{v}$	eigenvector
$\mathbf{V}$	similarity/transformation matrix of eigen- and principal (generalised) vectors
$\mathbf{w}$	principal vector
x, y, z	global Cartesian coordinate system at the zeroth nodal location
α	pre-twist angle per cell
γ	shear angle
ε	direct strain
θ	(torsional) rotation about the x-axis
κ	shear coefficient
λ	decay factor, eigenvalue
ν	Poisson's ratio
ψ	cross-sectional rotation

cell, together with knowledge of the boundary conditions. Straight (prismatic) repetitive one-dimensional (beam-like) structures have previously been analysed by Stephen and Wang (1996) as an eigenproblem for a state vector transfer matrix. The state vectors $\mathbf{s}_\mathrm{L}$ and $\mathbf{s}_\mathrm{R}$ consist of the nodal displacement and force components on the left- and right-hand sides, respectively, of the single cell of the repetitive structure, while the transfer matrix $\mathbf{G}$ is obtained through manipulation of the single cell stiffness matrix, $\mathbf{K}$. Non-unity eigenvalues of $\mathbf{G}$ occur as reciprocals, and describe the rate of decay of self-equilibrated end loading, as anticipated by Saint-Venant's principle. Multiple unity eigenvalues pertain to the transmission modes of tension, torsion, bending moment and shear, together with the rigid body displacements and rotations. From knowledge of the eigen- and principal vectors associated with the unity eigenvalues, equivalent continuum beam properties of cross-sectional area, Poisson's ratio, second moment of area, torsion constant and shear coefficient were calculated. The present paper extends this approach to pin-jointed structures