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The fundamental role of nonlocal and local balance laws of material forces in finite elastoplasticity and damage mechanics

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Abstract

In this paper the fundamental role of independent balance laws of material forces acting on dislocations and microdefects is shown. They enable a thermodynamically consistent formulation of dissipative deformation processes of continua with dislocation motion and defect evolution in the material space on meso- and microlevel.

The balance laws of material forces together with the classical balance laws of physical forces and couples, first and second laws of thermodynamics for physical and material space and general constitutive equations are the basis to develop a thermodynamically consistent framework of nonlocal finite elastoplasticity and brittle and ductile damage.

It is shown that a weakly-nonlocal formulation of the balance laws of material forces leads to gradient theories, where local theories are obtained, if all gradient contributions are assumed to be small. In this case the local balance laws of material forces together with the constitutive equations represent evolution laws of the material forces. In the classical approach of internal variables they are assumed from the outset with the result that there is a large number of different propositions in the literature.

The well-known splitting test of a circular cylinder of concrete is simulated numerically, where the process of deformation in the physical space and defect and plastic evolution in the material space is represented.

Keywords: Brittle damage; Ductile damage; Gradient damage; Gradient elastoplasticity; Material forces; Configurational forces; Dissipative processes

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1. Introduction

Motion and deformation of continua in the physical space are well defined within the concept of classical continuum mechanics and, if temperature changes are taken into account, within classical continuum thermodynamics. They are based on the postulates of physical linear and angular momentum and first and second law of thermodynamics supplemented by appropriate constitutive equations subject to objectivity requirements (Truesdell and Noll, 1965). In general, along with the deformation of the material body in the physical space a change of the internal material structure can occur accompanied by degradation of material properties. These changes are caused by physical mechanisms in the material space on macro-, meso-, and microlevel, respectively, where microcracks, microvoids and dislocations are observed in brittle and ductile material, respectively.

An appropriate description of nucleation and evolution of microdefects and their coupling with the deformation of the body in the physical space is not at all an easy task. Most concepts proposed so far in the literature of elastoplasticity and damage are internal variable approaches (see e.g. Kachanov, 1986; Chaboche, 1988; Hansen and Schreyer, 1994; Krajcinovic, 1996; Houlsby and Puzrin, 2000). Internal variable approaches are based on classical continuum mechanics to describe the macrodeformation, while the nucleation and evolution of microdefects are characterized by internal variables, for which evolution laws are postulated. Essential drawbacks of these theories are:

- Internal variable approaches are local theories, which are not able to describe appropriately localization phenomena (e.g. Bažant, 1991) leading to a pathological mesh-dependency of corresponding FE-solutions (see e.g. Miehe, 1998; Schieck et al., 1999);
- Local theories are not able to simulate length scale dependent problems (see e.g. Bažant and Ožbolt, 1990; Bažant and Planas, 1998);
- There is no recipe, how to chose evolution laws. Therefore, there are quite different propositions in the literature.

To overcome the numerical difficulties associated with the analysis of localization phenomena applying local theories integral and gradient enhancements were proposed in the literature (e.g. Aifantis, 1984; Bažant and Pijaudier-Cabot, 1988; de Borst and Mühlhaus, 1992; de Borst et al., 1995; de Borst, 2001), where the gradient enhancements can be obtained from integral enhancements by Taylor expansion restricting to the first term of the series. Gradient theories are weakly-nonlocal, but with the advantage that they are better accessible to numerical applications than integral enhanced theories. Fully nonlocal theories can be established also by taking into account the manifold structure of the material space with torsion and curvature (see e.g. Shizawa and Zbib, 1999; Cleja-Tigoiu, 2002a,b; Rakotomanana, 2004; Clayton et al., in press).

While the question of locality or nonlocality of the model is essential for the numerical analysis of localization phenomena as shearbands and damage concentrations, it does not explain the large number of proposed theories in finite elastoplasticity and damage mechanics and many controversial discussions in the literature on these fields. The crucial point and reason for this is the fact that in general to describe the dissipative process of defect evolution on meso- and mirolevel there are less equations than unknowns and therefore the missing equations are freely assumed as in the case of evolution laws.

Since Leibfried (1949), Peach and Koehler (1950) and Eshelby (1951) it is known that there are forces acting on microdefects as microcracks, voids and dislocations, and these forces are non-standard and called "material forces" and "configurational forces", respectively, contrary to the Newtonian "physical forces" acting on masses in the physical space. The material forces have to satisfy their own dynamical balance laws, which are independent of the balance laws of physical forces, coupled only by constitutive equations.