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An implicit consistent algorithm for the integration of thermoviscoplastic constitutive equations in adiabatic conditions and finite deformations

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Abstract

The so-called viscoplastic consistency model, proposed by Wang, Sluys and de Borst, is extended here to the integration of a thermoviscoplastic constitutive equation for J_2 plasticity and adiabatic conditions. The consistency condition in this case includes not only strain rate but also the effect of temperature on the yield function. Using the backward Euler integration scheme to integrate the constitutive equations, an implicit algorithm is proposed, leading to generalized expressions of the classical return mapping algorithm for J_2 plasticity, both for the iterative calculation of the plastic multiplier increment and for the consistent tangent operator when strain rate and temperature are considered also as state variables of the hardening equation. The model was implemented in a commercial finite element code and its performance is demonstrated with the numerical simulation of four Taylor impact tests.

Keywords: Finite element solution; Viscoplastic; Thermal softening; Impact; Finite deformation

1. Introduction

Many advanced processes in engineering such as high-speed metal forming (Rojek et al., 2001) and cutting (Molinari et al., 2002; Bäker et al., 2002), structures under crashes (Reyes et al., 2002), high-speed impact on metallic armours (Yadav et al., 2001; Rosenberg et al., 2004) and others, involve complex thermomechanical and multiaxial loading conditions which include large strain, high strain rates,

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Notations

c_v	specific heat
C	linear isotropic elastic tensor
d	rate of deformation tensor
F	deformation gradient tensor
l	velocity gradient tensor
r	plastic strain rate direction
s	deviatoric stress tensor
α	coefficient of thermal expansion
$\bar{\varepsilon}^{\mathrm{p}}$	equivalent plastic strain
η	Quinney–Taylor coefficient
ρ	material density
σ	Cauchy stress tensor
$\bar{\sigma}$	equivalent stress
σ_{Y}	yield stress
θ	temperature

temperature softening, adiabatic processes, etc. The numerical simulation of these phenomena requires the integration of the constitutive equations of the material, accounting for thermoviscoplastic hardening relations such as those proposed by Johnson and Cook (1983), Bodner et al. (1975), Zerilli et al. (1987), Litonski (1977) or the more recent equation by Rusinek and Klepaczko (2001). Two major kinds of model can be formulated to account for viscoplastic behaviour of materials: the *overstress models* (such as Perzyna (1966) and Duvaut and Lions (1972)) and the so-called *consistency model*, first proposed by Wang (1997) and Wang et al. (1997) and used by others (Ristinmaa et al., 2000; Winnicki et al., 2001; Heeres, 2001). Using the overstress models, the consistency condition is not fulfilled and stress states outside the yield surface are allowed so the Kuhn–Tucker conditions are not applicable. On the other hand, in the second approach the consistency condition for the yield function is enforced to include rate effects i.e.

$$f(\sigma, \vec{\kappa}, \dot{\vec{\kappa}}) = 0 \quad \text{at } \dot{\lambda} > 0 \tag{1}$$

$\vec{\kappa}$ being a vector including all the state variables and λ the plastic multiplier.

To integrate the set of non-linear thermoviscoplastic constitutive equations into the finite element method, two main tasks must be accomplished at level of the material point. The first one concerns the update of stress and state variables, driven by the strain increment. The second is related to the proper construction of the tangent stiffness used in global implicit FE algorithms, since the quadratic rate of convergence can be preserved only if a consistent (algorithmic) material stiffness is adopted (Simo et al., 1985; Ju, 1990).

In this paper, the consistency model is extended to integrate the thermoviscoplastic constitutive equations for adiabatic conditions and finite deformations. Using the backward Euler scheme, this consistency model provides a fully implicit numerical algorithm similar to the closest point projection commonly used for rate-independent problems on account of their robustness and stability. The update of stress and state variables is achieved through the solution of a single scalar non-linear equation in the plastic multiplier increment $\Delta\lambda$. Then the consistent tangent operator is determined by a systematic linearization of the corresponding algorithm.

The proposed scheme was implemented in the finite element commercial code ABAQUS/Explicit (2003) and its performance is demonstrated through the numerical simulation of four Taylor impact tests.