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An analytical and numerical approach for calculating effective material coefficients of piezoelectric fiber composites

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Abstract

The present work deals with the modeling of 1–3 periodic composites made of piezoceramic (PZT) fibers embedded in a soft non-piezoelectric matrix (polymer). We especially focus on predicting the effective coefficients of periodic transversely isotropic piezoelectric fiber composites using representative volume element method (unit cell method). In this paper the focus is on square arrangements of cylindrical fibers in the composite. Two ways for calculating the effective coefficients are presented, an analytical and a numerical approach. The analytical solution is based on the asymptotic homogenization method (AHM) and for the numerical approach the finite element method (FEM) is used. Special attention is given on definition of appropriate boundary conditions for the unit cell to ensure periodicity. With the two introduced methods the effective coefficients were calculated for different fiber volume fractions. Finally the results are compared and discussed.

Keywords: Composite materials; Piezoelectric; Micromechanics; Homogenization; Finite element method

1. Introduction

Piezoelectric materials have the property of converting electrical energy into mechanical energy and vice versa. This reciprocity in the energy conversion makes piezoelectric ceramics such as PZT (lead zirconium

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titanat) very attractive materials towards sensors and actuators applications. Even if their properties make them interesting, they are often limited, first by their weight, that can be a clear disadvantage for shape control and as a consequence, by their high specific acoustic impedance, which reduces their acoustic matching with the external fluid domain. Bulk piezoelectric materials have several drawbacks, hence composite materials are often a better technological solution in the case of a lot of applications such as ultrasonic transducers, medical imaging, sensors, actuators and damping. For the last 20 years, composite piezoelectric materials have been developed by combining piezoceramics with passive non-piezoelectric polymers. Superior properties have been achieved by these composites by taking advantage of most profitable properties of each constituents and a great variety of structures have been produced. Recently, due to the miniaturization of piezoelectric composites and the use of PZT fibers instead of piezoelectric bars, new applications toward electromechanical sensors and actuators have become possible. But, because the fibers are now much smaller than the wavelength, homogenization techniques are necessary to describe the behavior of piezoelectric composites.

Even if analytical and semi analytical models have been developed to homogenize piezoelectric composites, they are often reduced to specific cases. Numerical methods, such as the finite element method, seem to be a well-suited approach to describe the behavior of these materials, because there are no restrictions to the geometry, the material properties, the number of phases in the piezoelectric composite, and the size. However, finite element results are sensitive to mesh density. So it could be a difficult task to find appropriate meshes.

The prediction of the mechanical and electrical properties of piezoelectric fiber composites became an active research area in recent years. Except from experimental investigations, either micro- or macro mechanical methods are used to obtain the overall properties of piezoelectric fiber composites. Micro mechanical methods provide an overall behavior of piezoelectric fiber composites from known properties of their constituents (fiber and matrix) through an analysis of a periodic representative volume element (RVE) or a unit cell model. In the macro mechanical approach, on the other hand, the heterogeneous structure of the composite is replaced by a homogeneous medium with anisotropic properties. The advantage of the micro mechanical approach is that not only the global properties of the composites can be calculated, but also various mechanisms such as damage initiation and propagation, crack growth, etc. can be studied through the analysis.

A number of methods have been developed to predict and to simulate the coupled piezoelectric and mechanical behavior of composites. Basic analytical approaches have been reported (e.g. Chan and Unsworth, 1989; Smith and Auld, 1991), which are not capable of predicting the response to general loading, i.e. they do not give the full set of overall material parameters. Semi analytical and Hashin/Shtrikman-type bounds for describing the complete overall behavior (i.e. all elements of the material tensors) have been developed (Bisegna and Luciano, 1996, 1997) which are useful tools for theoretical considerations. However, the range between the bounds can be very wide for certain effective moduli. Mechanical mean field type methods have been extended to include electro-elastic effects (Benveniste, 1993; Dunn and Taya, 1993; Wang, 1992; Chen, 1993) based on an Eshelby-type solution for a single inclusion in an infinite matrix (Benveniste, 1992; Dunn and Wienecke, 1997). Such mean field type methods are capable of predicting the entire behavior under arbitrary loads. However, they use averaged representations of the electric and mechanical field within the constituents of the composite. This restriction can be overcome by employing periodic micro field approaches (commonly referred to as unit cell models) where the fields are typically solved numerically with high resolution, e.g. by the finite element method (Gaudenzi, 1997). In such models the representative unit cell and the boundary conditions are designed to capture a few special load cases, which are connected to specific deformation patterns (e.g. Brockenbrough and Suresh, 1990; Böhm, 1993; Gunawardena et al., 1993; Cleveringa et al., 1997). This allows the prediction of only a few key material parameters; for example, only normal loads can be applied consistently using the symmetry boundary conditions. A different method which can handle arbitrary loading scenarios is the