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Evaluation of multi-modes for finite-element models: systems tuned into 1:2 internal resonance

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Abstract

A non-linear multi-mode of vibration arises from the coupling of two or more normal modes of a non-linear system under free-vibration. The ensuing motion takes place on a $2M$-dimensional invariant manifold in the phase space of the system, M being the number of coupled linear modes; the manifold contains a stable equilibrium point of interest, and at that point is tangent to the $2M$-dimensional eigenspace of the system linearised about that equilibrium point, which characterises the corresponding M linear modes. On this manifold, M pairs of state variables govern the dynamics of the system; that is, the system behaves like an M-degree-of-freedom oscillator. Non-linear multi-modes may therefore come about when the system exhibits non-linear coupling among generalised co-ordinates. That is the case, for instance, of internal resonance of the 1:2 or 1:3 types, for systems with quadratic or cubic non-linearities, respectively, in which a four-dimensional manifold should be determined. Evaluation of non-linear multi-modes poses huge computational challenges, which is the explanation for very limited reports on the subject in the literature so far. The authors developed a procedure to determine the non-linear multi-modes for finite-element models of plane frames, using the method of multiple scales. This paper refers to the case of quadratic non-linearities. The results obtained by the proposed technique are in good agreement with those coming out from direct integration of the equations of motion in the time domain and also with those few available in the literature.

Keywords: Non-linear dynamics; Non-linear modes; Non-linear multi-modes; Quadratic non-linearities; Finite-element model; Reduction technique

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1. Introduction

According to Boivin et al. (1994), Boivin et al. (1995a) and Boivin et al. (1995b), non-linear multi-modes of vibration can be understood as an extension of the non-linear normal modes, in the case two or more of them interact. Such interactions are stronger in presence of internal resonance. The ensuing free vibration motion takes place in an invariant manifold embedded in the phase space, whose dimension is twice the number of the normal modes which interact. This manifold contains a stable equilibrium point, and is tangent there to the sub-eigenspace of the linearised system, which characterises the corresponding linear modes. The multi-mode can be locally described by a linear combination of the linear modes. On this manifold, the system behaves like an M-degree-of-freedom oscillator, where M is the number of coupled normal modes.

Both non-linear normal modes and multi-modes may be efficient projection functions to be used in the reduction of degrees of freedom of non-linear systems, according to Mazzilli et al. (2001).

The authors developed a technique based on the method of multiple scales to evaluate the non-linear multi-modes of discrete systems whose equations of motion are of the form of (1). Such a technique is an extension of the procedure already proposed by them with success to the evaluation of non-linear normal modes, see Mazzilli and Baracho Neto (2002).

For simplicity and conciseness in the presentation, this paper will consider only systems in which the quadratic non-linearities are the important ones, such that there are only two coupled normal modes in internal resonance, their linear frequencies being in the 1:2 ratio. For systems with cubic non-linearities and internal resonance of the 1:3 type, reference is made to Baracho Neto and Mazzilli (2005).

Suppose the equations of motion for a non-linear system with n degrees of freedom are of the form (1), which is sufficiently general to accommodate not only systems that are naturally discrete, but also those discretised by the finite-element method. That is, for instance, the case of planar frames, see Mazzilli and Baracho Neto (2002).

$$M_{rs}\ddot{p}_s + D_{rs}\dot{p}_s + U_{,r} = \mathscr{F}_r \tag{1}$$

$$\begin{aligned} M_{rs} &= {}^0M_{rs} + {}^1M^i_{rs}p_i + {}^2M^{ij}_{rs}p_ip_j \\ D_{rs} &= {}^0D_{rs} + {}^1D^i_{rs}\dot{p}_i + {}^2D^{ij}_{rs}\dot{p}_ip_j \\ U_{,r} &= {}^0K_{rs}p_s + {}^1K^i_{rs}p_ip_s + {}^2K^{ij}_{rs}p_ip_jp_s \end{aligned} \tag{2}$$

$[M]$ and $[D]$ are, respectively, the matrices of mass and equivalent viscous damping, and $\{U\}$ and $\{F\}$ are, respectively, the elastic force and the applied load vectors. This latter one is usually null in modal analysis, unless vibration about the deformed equilibrium configuration is being considered, in which case it is a static load vector. The generalised co-ordinates, velocities and accelerations are written as p_i, $\dot{p}_i$ and $\ddot{p}_i$. Einstein's notation is being used throughout the text, so that repeated indices mean summation from 1 to n, unless otherwise stated. Note that system (1) comprises both quadratic and cubic non-linearities, although, in the present study, only the quadratic non-linearities are assumed to be the important ones.

2. Non-linear multi-modes for 1:2 internal resonance: time response

Non-linear multi-modes of systems with quadratic non-linearities tuned into 1:2 internal resonance are now sought with the help of the method of multiple scales. The output will be the time response of each generalised co-ordinate in the form of an asymptotic expansion of a small positive non-dimensional perturbation parameter ε. Note that the real time t is replaced by several time scales T_k, as defined below.