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Stabilization of beam parametric vibrations with shear deformations and rotary inertia effects

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Abstract

The purpose of this theoretical work is to present a stabilization problem of beam with shear deformations and rotary inertia effects. A velocity feedback and particular polarization profiles of piezoelectric sensors and actuators are introduced. The structure is described by partial differential equations with time-dependent coefficient including transverse and rotary inertia terms, general deformation state with interlaminar shear strains. The first order deformation theory is utilized to investigate beam vibrations. The beam motion is described by the transverse displacement and the slope. The almost sure stochastic stability criteria of the beam equilibrium are derived using the Liapunov direct method. If the axial force is described by the stationary and continuous with probability one process the classic differentiation rule can be applied to calculate the time-derivative of functional. The particular problem of beam stabilization due to the Gaussian and harmonic forces is analyzed in details. The influence of the shear deformations, rotary inertia effects and the gain factors on dynamic stability regions is shown.

Keywords: Vibration control; Piezoelectric actuators; Shear deformations; Rotary inertia; Liapunov method

1. Introduction

In the paper, theoretical fundamentals of stabilization of beam with shear deformations and rotary inertia effects are presented. The piezoelectric layers are glued to the both sides of the beam compressed by time-dependent axial forces. A velocity feedback and particular polarization profiles of piezoelectric sensors and actuators are introduced. The structure is described by partial differential equations including

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transverse and rotary inertia terms, general deformation state with interlaminar shear strains. A viscous model of external damping with the constant proportionality coefficient is assumed to describe a dissipation of the structure energy both in the transverse and rotary motion. The first order deformation theory is utilized to investigate beam vibrations. The beam motion is described by the transverse displacement. In order to estimate deviations of solutions from the plane equilibrium state a scalar measure of distance equal to the square root of the suitable functional is introduced. The almost sure stochastic stability criteria of the structure equilibrium are derived using the Liapunov direct method. If the axial force is described by the stationary and continuous with probability one process the classic differentiation rule can be applied to calculate the time-derivative of functional. In order to find an exponential one-side estimation the calculus of variation is used. The associated Euler equations in the form of system of differential equations are solved analytically and the stabilization problem is reduced to transcendental algebraic inequality with respect the exponent of estimation. The particular problem of beam stabilization due to the Gaussian and harmonic forces is analyzed in details. The influence of the shear deformations, rotary inertia effects and the gain factors on dynamic instability regions is shown.

The problem of correction of shear on the transverse beam vibration goes back to the work of Timoshenko in 1921 (see Timoshenko and Gere, 1961). The influence of shear deformation on the natural frequencies of laminated rectangular plates was examined by Dave and Craig (1985). Shear deformation effects on thermal buckling of cross-ply composite laminates were analyzed by Mannini (1997). Timoshenko beam-bending solutions in terms of Euler–Bernoulli solutions were given by Wang (1995). The influence of transverse shear on dynamic stability domains was studied by Pavlović et al. (2001). The thermally induced parametric vibrations of laminated plates with shear effects due to the time-dependent temperature with Gaussian and harmonic distributions were analyzed (Tylikowski, 2003). Analytical solutions for the length and position of strain-induced patch actuators for the static adjustments of Timoshenko's beam deflection were presented by Ang et al. (2000). The effects of the feedback control gain on the parametric vibrations of a beam with piezoelectric layers compressed by harmonic axial force were examined by Chen et al. (2002).

2. Basic assumptions, definitions

Consider the beam of length l, width b, and thickness h_b, loaded by axial time-dependent force with piezoelectric layers mounted on each of two opposite sides. The beam is simply supported on both ends. The piezoelectric layers are assumed to be bonded on the beam surfaces and the mechanical properties of the bonding material are represented by the effective damping coefficient calculated from the rule of mixture. The damping coefficient is a linear function of both the beam and bonding layer damping coefficients. It is assumed that the transverse motion dominates the axial vibrations. The thickness of the actuator and the sensor is denoted by h_a and h_s, respectively. Assuming a negligible stiffness of the piezolayer in comparison with that of the beam and the changing width $b_s(X)$ of the sensor, the changing width $b_a(X)$ of the actuator the influence of the piezoelectric actuator on the beam can be reduced to bending moment M_x distributed along the actuator.

The transverse motion of the beam is described by the uniform equation with time-dependent coefficients. Its trivial solution $w = 0$ corresponds to the undisturbed state. The trivial solution is called stable in Liapunov sense if the following definition is satisfied:

$$\bigwedge_{\varepsilon>0} \bigvee_{\delta>0} \|w(0,.)\| < \delta \Rightarrow \bigwedge_{t>0} \|w(t,.)\| < \varepsilon \tag{1}$$

where $\|\cdot\|$ is a measure of distance of disturbed solution w from the equilibrium state. When the axial force is a stochastic processes we call the trivial solution almost sure asymptotically stable if