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An embedding method for modeling micromechanical behavior and macroscopic properties of composite materials

J. Wang [a,*], S.G. Mogilevskaya [b], S.L. Crouch [b]

[a] *Department of Civil and Coastal Engineering, University of Florida, 365 Weil Hall, P.O. Box 116580, Gainesville, FL 32611, USA*
[b] *Department of Civil Engineering, University of Minnesota, 500 Pillsbury Drive S.E., Minneapolis, MN 55455, USA*

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Abstract

This paper presents a numerical method for modeling the micromechanical behavior and macroscopic properties of fiber-reinforced composites and perforated materials. The material is modeled by a finite rectangular domain containing multiple circular holes and elastic inclusions. The rectangular domain is assumed to be embedded within a larger circular domain with fictitious boundary loading represented by truncated Fourier series. The analytical solution for the complementary problem of a circular domain containing holes and inclusions is obtained by using a combination of the series expansion technique with a direct boundary integral method. The boundary conditions on the physical external boundary are satisfied by adopting an overspecification technique based on a least squares approximation. All of the integrals arising in the method can be evaluated analytically. As a result, the elastic fields and effective properties can be expressed explicitly in terms of the coefficients in the series expansions. Several numerical experiments are conducted to verify the accuracy and efficiency of the numerical method and to demonstrate its application in determination of the macroscopic properties of composite materials.

Keywords: Embedding method; Fiber-reinforced composites; Perforated materials; Effective properties; Direct boundary integral method; Fourier series; Least squares

* Corresponding author. Tel.: +1 352 392 9537; fax: +1 352 392 3394.
E-mail address: jwang@ce.ufl.edu (J. Wang).

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1. Introduction

Fiber-reinforced composites and perforated materials are widely used in engineering structures. Some of these materials (for example, unidirectional fiber-reinforced composites, or thin-plate solids consisting of disks or holes) can be represented as two-dimensional linearly elastic solids containing multiple holes and locally isotropic inclusions. The inclusions refer to the fibers or disks in composite materials and the holes are either due to the inherent porosity of the material or a result of the manufacturing process (i.e. caused by drilling). The holes and inclusions cause significant stress disturbance, especially when they are closely spaced, leading to a considerable change of the macroscopic properties as well as the micromechanical behavior of the material. Thus, it is of engineering importance to calculate the elastic fields and effective properties of a two-dimensional solid containing multiple holes and inclusions. In the work presented in this paper, we consider the particular but practically important case in which the holes and inclusions are circular.

This paper presents an extension of the previous work of the authors on modeling composite materials using a direct boundary integral approach, which was first presented by Mogilevskaya and Crouch (2001) for multiple circular inclusions in an infinite plane. This approach was later extended to include circular holes as well as cracks (e.g. Wang et al., 2001, 2003a,c) and to consider a finite domain with a circular boundary (Wang et al., 2003c). These analyses combine the series expansion technique with a direct boundary integral method in which complex singular and hypersingular integrals are written directly in terms of the actual boundary tractions, displacements, and—for cracks—displacement discontinuities (Linkov and Mogilevskaya, 1994). The unknown boundary parameters are expressed globally in terms of series expansions of orthogonal functions (i.e. Fourier series for circular boundaries and Chebyshev polynomials for cracks). All the integrals involved in the analyses are evaluated analytically and numerical errors only come from truncation of the series. After incorporating a fast multipole algorithm, the approach is capable of solving large scale practical problems involving thousands of objects (e.g. Wang et al., in press). Because of its analytic nature, the approach has advantages in accuracy and efficiency over some other numerical methods [for example, the series expansion method (Isida and Sato, 1984; Wang et al., 2000), the finite element method (Meguid, 1986; Wacker et al., 1998), and the standard boundary element method (Eischen and Torquato, 1993; Greengard and Helsing, 1998; Liu et al., 2000; Kong et al., 2002)] for the particular problem under consideration.

In this paper, we consider a finite domain of rectangular or square shape. Such a model can be used to design laboratory experiments for direct measurement of micromechanical behavior and macroscopic properties of composite materials. For a domain of such shape, however, the problem can no longer be solved analytically. In order to retain the main features of our approach, we suggest a simple embedding technique that is well tailored for this particular type of problem. Instead of solving the problem directly, we embed the physical solution domain into a circular disc and apply fictitious loading on the boundary so as to satisfy the prescribed conditions on the physical external boundary. The subsequent analysis is based on the analytical solution we have obtained for multiple circular inclusions and holes in a finite circular domain (Wang et al., 2003c).

The term "embedding method" is used in the literature to describe different numerical techniques, all of which consider the physical solution domain as a part of a larger fictitious domain with simple geometry. In one of these approaches by Boley and collaborators (Boley, 1961; Boley and Yagoda, 1971), the fictitious boundary conditions on the artificial boundary are found by satisfying the conditions on the real physical boundary. In our approach, we use the term "embedding method" in the same sense as Boley (1961) and Boley and Yagoda (1971) with the embedding domain chosen to be circular. Following Wang et al. (2003c), the fictitious tractions and displacements on the boundary of the embedding domain and other unknown boundary functions are approximated by truncated complex Fourier series. As a result, all the integrals involved in the analysis are evaluated analytically. The stress and displacement fields everywhere inside the