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Predictions of bifurcation and instabilities during dynamic extension

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Abstract

Dynamic bifurcation and flow instabilities of cylindrical bars, made of an incompressible strain hardening plastic material, are investigated. A Lagrangian linear perturbation analysis is performed to obtain a fourth order partial differential equation which governs the evolution of the perturbation. The analysis shows that inertia slows down the growth of long wavelengths while bidimensional effects conjugated to strain hardening extinct short wavelengths. The present approach is applied successfully to the analysis of bifurcation and instabilities in (i) a rectangular block during plane strain extension, (ii) a circular bar during uniaxial extension. New results are obtained in the case of rate independent materials and a synthetical point of view is obtained for rate dependent behaviors.

Keywords: Bifurcation; Instability; Dynamic extension

1. Introduction

In rapid stretching, structures can develop a multiple necking pattern which leads to the fracture in several fragments. Experimental evidence of this phenomenon has been reported by several authors. Niordson (1965) has developed an experimental device in which an intense electromagnetic field is used to expand thin rings at high strain rate. In the loaded specimen, many necks are observed along the circumference. Grady and Benson (1983) performed dynamic expansion of aluminium and copper rings using the former technique. The authors enlight the enhanced ductility of metals in dynamic conditions compared to quasi-static conditions. They observed also the fragmentation of rings at high velocity testings. They have noted that the number of fragments increases with the loading velocity. More recently, Altynova et al. (1996) have also performed expansion of rings (Al, T6AL, Cu alloys) by electromagnetic means. Trends observed by Grady and Benson (1983) are retrieved by these authors. The fragmentation is also observed in axisymmetric jet formed by the collapse of a linear shaped liner under explosive loading. During the flight,

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the jet is stretched at high strain rate and fragments are sometimes observed (see Karpp and Simon, 1976; Chou et al., 1977).

Many contributions have been devoted to the analysis of bifurcation and instabilities occuring during plastic loading. Most of them were concerned by quasi-static situations. Hill and Hutchinson (1975) have developed a quasi-static bifurcation analysis for a rectangular plate subjected to plane strain tension. The material has a rate independent behavior. Depending on the deformation state, the bifurcation can occur in the elliptic, parabolic or hyperbolic regimes. Young (1976) carried out a similar analysis in plane strain compression. Benallal and Tvergaard (1995) examined the role of non local effects on bifurcation in the plane strain tension and compression tests.

Multiple necking during high strain rate loadings is the result of inertia effects. Therefore the above mentioned analyses have to be extended to account for inertia forces. Sorensen and Freund (1998) have extended the approach of Hill and Hutchinson (1975) in dynamic conditions. Acceleration term in the momentum balance is taken into account. But the hydrostatic pressure contribution resulting from lateral inertia is ignored so that this dynamic analysis is valid for large ratio of length to width of the block. The material is rate independent and the incompressibility assumption is adopted. The elliptic, hyperbolic and parabolic regimes are identical to those established by Hill and Hutchinson (1975). Owing to the inertia term in the momentum balance, the rate of growth of the bifurcation mode is evaluated. It is found that long wavelength modes are suppressed by inertia. Shenoy and Freund (1999) improved the previous work by taking into account of the hydrostatic pressure contribution due to the lateral inertia. In this analysis, the material behavior is rate independent with an isotropic hardening. By considering the rate of growth of perturbations, it is observed that a particular wavelength is selected which characterizes the size of the fragments. The authors claim that inertia is responsible for this phenomenon since they considered that inertia suppress both short and long wavelength mode of bifurcation. In addition the authors have enlightened the fact that the number of necks is not influenced by the level of strain hardening.

Ring experiments have also been modelled by finite element calculations. An interesting work has been performed by Han and Tvergaard (1995). The material is a rate independent elastic–plastic solid. A small imperfection triggers the onset of necking. Nevertheless, due to wave propagation, the number of necks can exceed the number of initial thin points introduced by the imperfection. The authors have shown that the neck spacing is dependent on the loading and on the aspect ratio of the cross section. On the contrary, the magnitude of the initial defect and of the strain hardening coefficient does not influence the necking pattern.

The previous works concern rate-independent material. For rate dependent material, the problem of a rectangular block subjected to tension has been analysed by Hutchinson et al. (1978). Using a linear perturbation analysis, the authors have concluded that the strain rate sensitivity effects damp short wavelengths. The effect of strain rate sensitivity has been already mentioned by Hutchinson and Neale (1977) in the long wavelength analysis of neck formation in a viscoplastic bar. To model fragmentation in viscoplastic solids, Fressengeas and Molinari (1994) have extended the previous work by adding inertial effects. It was demonstrated that inertia slows down the growth of long wavelengths. This role in combination with the stabilizing aspects of viscosity and of bidimensional effects on short wavelengths leads to the selection of an intermediate wavelength (the fastest growing mode). Note that the fastest growing mode is time dependent. The role of inertia was already mentioned by Fressengeas and Molinari (1985). An extension of the theory proposed by Fressengeas and Molinari (1994) has been carried out by Jeanclaude and Fressengeas (1997). They analysed the fragmentation of a rapidly stretching bar in an axisymmetric loading. This bidimensional dynamic analysis has provided similar results (selection of an intermediate wavelength due to inertia and strain rate sensitivity).

In this paper, a theoretical analysis of dynamic bifurcation of a cylindrical bar is performed. The material is incompressible rate insensitive with strain hardening. Owing to a linear perturbation analysis, the rate of growth of the perturbation is evaluated. Various stabilizing effects delay the growth of disturbances. Inertia slows down the long wavelengths whereas bidimensional effects damp the short wavelengths.