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A crack bridging model for bonded plates subjected to tension and bending

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Abstract

A crack bridging model is presented for analysing the tensile stretching and bending of a cracked plate with a patch bonded on one side, accounting for the effect of out-of-plane bending induced by load-path eccentricity inherent to one-sided repairs. The model is formulated using both Kirchhoff–Poisson plate bending theory and Reissner's shear deformation theory, within the frameworks of geometrically linear and nonlinear elasticity. The bonded patch is represented as distributed springs bridging the crack faces. The springs have both tension and bending resistances; their stiffness constants are determined from a one-dimensional analysis for a single strap joint, representative of the load transfer from the cracked plate to the bonded patch. The resulting coupled integral equations are solved using a Galerkin method, and the results are compared with three-dimensional finite element solutions. It is found that the formulation based on Reissner's plate theory provides better agreement with finite element results than the classical plate theory.

Nomenclature

a	half crack length
c_{ij}	elements of spring compliance matrix ($i = 1, 2$)
E	Young's modulus
G	strain energy release rate
h_1	normalised crack opening displacement (u/a)
h_2	normalised generalised crack face rotation displacement ($\frac{1}{6}\theta t_P/\alpha$)
I	moment of inertia
k_α	spring stiffness matrix ($\alpha = t$, b where t denotes tension and b denotes bending)
K	stress intensity factor
$K_b^{(K)}$	bending stress intensity based on Kirchhoff–Poisson plate theory

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$K_b^{(R)}$ bending stress intensity based on Reissner's plate bending theory
M bending moment
N membrane force
S stiffness ration $(E'_R t_R / E'_P t_P)$
t thickness
u crack face displacement (mid-plane)
$\bar{z}$ position of the neutral plane of a plate.

Greek symbols
θ crack face rotation
μ shear modulus
ν Poisson's ratio
σ_0 prospective stress at the crack location
σ_m membrane stress
σ_b maximum bending stress.

Subscripts
P, R, A denoting parameters pertaining to the plate, the reinforcement, and the adhesive layer.

Superscripts
∞ parameters pertaining to the remote loading.

1. Introduction

With the increasing use of bonded repair techniques for repairing cracks and other damages in primary airframe structures, the design and evaluation of bonded repairs is becoming a major concern to meet certification requirements of damage tolerant repairs to critical structures (Baker, 1997). A critical repair is defined as one in which the residual strength of the unrepaired component would be lower than the design ultimate or, even worse, the design limit load. To certify critical repairs to primary structures we need to be able to demonstrate by analysis and/or test that the repair can meet the residual strength and damage tolerance requirements. To ensure adequate damage tolerance of the repaired structure, it is essential to determine the reduction in the stress intensity factor after repair so as to ensure that (i) the residual strength has been restored to an acceptable level, and (ii) the growth rate of the crack under fatigue condition is sufficiently slow to ensure an acceptable residual life, or inspection interval.

A bonded repair, which involves adhesively bonding a composite patch to a cracked or damaged structure, may fail in a number of modes, such as patch failure, failure of the adhesive layer, failure of the plate outside the repair region due to stress elevation near the termination of the patches, or insufficient reduction in the stress intensity factor of the crack thus leading to continuous crack growth. Over the past two decades analytical procedures (Erdogan and Arin, 1972; Keer et al., 1976; Rose, 1981, 1982, 1988) have been developed to address these issues, assuming that the bonded structure is under tension only and there is no secondary bending. In particular, the simple, closed form solution derived by Rose (1981, 1982, 1988) for the limiting value of the stress intensity factor has played a key role in the development of bonded repair methodology, which has been