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The anisotropic material constitutive models for the human cornea

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Abstract

This paper presents an anisotropic analysis model for the human cornea. The model is based on the assumption that the fibrils in the cornea are organised into lamellae, which may have preferential orientation along the superior–inferior and nasal-temporal directions, while the alignment of lamellae with different orientations is assumed to be random. Hence, the cornea can be regarded as a laminated composite shell. The constitutive equation describing the relationships between membrane forces, bending moments, and membrane strains, bending curvatures are derived. The influences of lamella orientations and the random alignment of lamellae on the stiffness coefficients of the constitutive equation are discussed.

Keywords: Cornea; Collagen fibrils; Anisotropy; Biomechanics; Constitutive model

1. Introduction

The mechanical properties of the human cornea are fundamental to our understanding of corneal behaviour in response to keratorefractive surgery and aspects of corneal physiology and physiological optics where mechanics plays an important role. Research on the whole corneal structure has showed that the mechanical behaviour of the cornea is rather complicated (Hjortdal, 1996; Pinsky and Datye, 1991; Shin et al., 1997). The unexpected non-uniform distribution of strains in the meridian of the cornea measured in experiments indicates that the cornea may have regionally different mechanical properties (Hjortdal, 1996; Shin et al., 1997). The principal causes for the regional variability in mechanical properties of the human cornea may include the variability of the elastic modulus of collagen fibrils, the reinforcing ratios and orientation of the fibrils, and the sequential recruitment of the fibrils. It has long been debated whether the collagen fibrils in the corneal stroma are randomly orientated or have preferential directions (Komai and Ushiki, 1991; Maurice, 1988). In the literature it was suggested that the collagen fibrils in the deeper layers of the corneal stroma are not isotropically arranged, but rather adopt a preferential orientation along the superior–inferior and nasal-temporal corneal meridians (Kokott, 1938). Indeed, recent X-ray scattering studies have confirmed this and indicated that the preferred orientation is more prevalent in the posterior half of the stroma (Aghamohammadzadeh et al., 2004; Boote et al., 2005; Meek and Newton, 1999), suggesting that the cornea exhibits anisotropic material behaviour. However, due to lack of quantitative information about the exact distribution of collagen fibrils in the human cornea and limbus, the anisotropic material model has never been implemented in the mechanical model of human corneas (Pinsky et al., 2005). Numerical modelling based on the finite element method has been used to predict the corneal response to various mechanical actions for many years. However, in most of existing models the cornea was still assumed as either an isotropic elastic material (Anderson et al., 2004; Hjortdal, 1996; Wollensak et al., 2003) or an axisymmetrically orthotropic elastic material (Pinsky and Datye, 1991; Shin et al., 1997). The purpose of the present paper is to investigate the influence of the orientation of collagen fibrils on the overall

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corneal mechanical behaviour and to derive the anisotropic material constitutive equations for the human cornea.

2. Material constitutive model

The human cornea is a soft biological tissue that consists of five layers lying parallel to its anterior surface. From outside to inside these layers are called the epithelium, Bowman's membrane, stroma, Descemet's membrane, and the endothelium (Maurice, 1984). The most important structural element in these five layers is the stroma, which forms about 90% of the corneal thickness and is made up of interlacing layers of collagen fibrils embedded in a highly hydrated proteoglycan-containing matrix (Maurice, 1988). Given the fact that the Young's modulus of the collagen fibrils in the fibril direction is of the order of 1.0 GPa (Fung, 1981; Jue and Maurice, 1986), whereas the Young's modulus of the ground substance is only of the order of 10^{-5} GPa (Hjortdal, 1996), the stroma will clearly exhibit strongly directional mechanical properties. Therefore, the response of the cornea is not weakly or mildly anisotropic, it is usually highly anisotropic. There has been strong evidence that for highly anisotropic composite materials isotropic theory would not provide even a rough approximation to their behaviour under most types of loading conditions (Rogers, 1984). It is therefore important to know how the orientation of collagen fibrils influences the overall corneal mechanical behaviour.

In the present study, the cornea is assumed to be a laminated composite shell made of a large number of unidirectionally reinforced thin lamellae placed in various different angles. According to the semi-empirical model developed by Halphin and Tsai (1969), the material constants describing the macromechanical behaviour of the single unidirectional lamella can be expressed in terms of the individual material constants of the fibrils and matrix together with the parameters describing the fibrils and packing geometries as follows:

$$E_1 \approx E_{\mathrm{f}} V_{\mathrm{f}}, \tag{1a}$$

$$E_2 \approx \frac{1+\xi V_{\mathrm{f}}}{1-V_{\mathrm{f}}} E_{\mathrm{m}}, \tag{1b}$$

$$G_{12} \approx \frac{1+\xi V_{\mathrm{f}}}{1-V_{\mathrm{f}}} G_{\mathrm{m}}, \tag{1c}$$

$$v_{12} \approx v_{\mathrm{f}} V_{\mathrm{f}} + v_{\mathrm{m}}(1-V_{\mathrm{f}}), \tag{1d}$$

$$v_{21} = v_{12}\frac{E_2}{E_1} \approx 0, \tag{1e}$$

where E_1 and E_2 are the Young's modulus of the lamella in the longitudinal and transverse directions, G_{12} is the in-plane shear modulus of the lamella, v_{12} and v_{21} are the major and minor Poisson's ratios of the lamella, E_{f} and v_{f} are the Young's modulus and Poisson's ratio of the fibrils, E_{m} and v_{m} are the Young's modulus and Poisson's ratio of the matrix, G_{m} is the shear modulus of the matrix, V_{f} is the volume fraction of the fibrils in the lamella, ξ is a reinforcing factor and its value depends on the geometries of fibrils and packing and the loading conditions. For a fibre geometry of circular fibres in a packing geometry of a square array, $\xi = 2$ (Halphin and Tsai, 1969). The in-plane stress–strain relation of the lamella thus can be expressed as,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} E_1 & v_{12}E_2 & 0 \\ v_{12}E_2 & E_2 & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \tag{2}$$

where σ_1, σ_2, and τ_{12} are the in-plane stress components, ε_1, ε_2, and γ_{12} are the corresponding strain components. The stress–strain relationship given in Eq. (2) is for the local orthogonal lamella coordinate system (1–2 axes). For the global Cartesian coordinate system (x-y axes) the stress–strain relationship can be obtained by using the transformation of stresses and strains, which is expressed as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3a}$$

in which,

$$Q_{11} = E_1c^4 + E_2s^4 + 2(v_{12}E_2 + 2G_{12})(sc)^2, \tag{3b}$$

$$Q_{12} = (E_1 + E_2 - 4G_{12})(sc)^2 + v_{12}E_2(s^4 + c^4), \tag{3c}$$

$$Q_{22} = E_1s^4 + E_2c^4 + 2(v_{12}E_2 + 2G_{12})(sc)^2, \tag{3d}$$

$$Q_{16} = (E_1 - v_{12}E_2 - 2G_{12})c^3s - (E_2 - v_{12}E_2 - 2G_{12})s^3c, \tag{3e}$$

$$Q_{26} = (E_1 - v_{12}E_2 - 2G_{12})s^3c - (E_2 - v_{12}E_2 - 2G_{12})c^3s, \tag{3f}$$

$$Q_{66} = (E_1 + E_2 - 2v_{12}E_2 - 2G_{12})(sc)^2 + G_{12}(s^4 + c^4), \tag{3g}$$

where $s = \sin\theta$, $c = \cos\theta$, and θ is the angle measured from x-axis rotating counterclockwise to 1-axis (see Fig. 1). The above stress–strain equation can be further extended to the standard constitutive equations for laminated composite shells as follows,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x^o \\ \kappa_y^o \\ \kappa_{xy}^o \end{Bmatrix} \tag{4a}$$

in which,

$$A_{ij} = \sum_{k=1}^{n} [(Q_{ij})]_k (h_k - h_{k-1}), \quad i = 1,2,6;\; j = 1,2,6, \tag{4b}$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} [(Q_{ij})]_k (h_k^2 - h_{k-1}^2), \quad i = 1,2,6;\; j = 1,2,6, \tag{4c}$$