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Analysis of tensegrity structures subject to dynamic loading using a Newmark approach

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ABSTRACT

Tensegrities are lightweight structures whose integrity is based on a balance between tension and compression. In general, these systems have low structural damping, and this leads to a challenge when they are subjected to dynamic loading. This paper presents an analysis for linearized dynamic tensegrity structures under dynamic loading near the equilibrium configuration using a Newmark method. Numerical examples are presented to demonstrate the effectiveness of the proposed scheme. The Newmark method is more efficient than other classical approaches to solve dynamical tensegrity structure models. Here, the effects of damping and slackening condition on dynamic analysis of tensegrity structures are also investigated.

1. Introduction

Tensegrity structures are a class of spatial structural systems composed of cable (in tension) and strut (in compression) components with reticulated connections, and assembled in a self-balanced fashion [1]. Their integrity is based on a balance between compression and tension. In the literature, there are several descriptions of tensegrity structures based on their intended focus [2,3]. Adequate prestress is essential for the stability of tensegrity structures.

The behavior of tensegrity systems under dynamic external loads is also widely studied. For the first time, Motro et al. [4] performed a dynamic experimental and numerical work on a tensegrity structure. They showed that a linearized dynamic model presents a good approximation in order to analyze nonlinear behavior of tensegrity structures around an equilibrium configuration. Furuya [5] expanded the concept of deployable tensegrity structures in space application. He also investigated the vibrational property of a tensegrity mast and demonstrated that the modal frequencies increase as the pre-stress increases. A double-layer tensegrity grid subjected to dynamic loading was experimentally studied by Kono et al. [6]. Kahla et al. [7] also studied the response of tensegrity system to dynamic loading, and developed the incremental equation of motion with respect to the configuration of the system using updated Lagrangian formulation. Murakami [8] proposed the basic equations for dynamic analyses tensegrity structures based on Eulerian and Lagragian formulation, and developed a numerical simulation and modal analysis of two and three dimensions tensegrity modules. Sultan et al. [9] utilized linearized dynamic models for two classes of tensegrity systems, and demonstrated, in general, modal dynamic increases with the pre-stress. Arsenault et al. [10] performed dynamic analyses of planar tensegrity modules with 1, 2 and 3 degrees of Freedom (DOF). Linearized dynamic model used to improve the dynamic control performance of a tensegrity system was studied by Masic et al. [11,12], although, they only optimized the pre-stress of the structure, and not its spatial configuration. Tan et al. [13] investigated the nonlinear vibration of a cable-stiffened pantographic deployable structure, and showed the natural frequencies of system depend on the level of cable pretension. Ali et al. [14] performed the dynamic analysis and vibration control of a full scale five module tensegrity structure. They showed experimentally and numerically that control of the self-stress influences the dynamic behavior. Faroughi and Mirats Tur [15] used linearized dynamic model to design tensegrity structures corresponding to vibration properties. Greco et al. [16] investigated the nonlinear geometrical behavior of a double layer tensegrity structure subject to dynamic loading in time domain. Cheong et al. [17] presented a numerical correction algorithm for execution of the dynamics of tensegrity systems described by non-minimal coordinates. More recently, Nagase and Skelton [18] derived the equation of motion

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in vector form for any class k tensegrity system dynamics. Seunghye and Lee [19] also investigated the force identification of pre-stress levels using the natural frequencies of cable-strut structures. Oliveto and Sivaselvan [20] proposed a complementary framework for dynamic analysis of a classical tensegrity grid without internal mechanisms in order to improve convergence of algorithms. However, their formulation was restricted to small-deformation analysis. Michielsen et al. [21] studied the steady-state dynamics of a base excited tensegrity module carrying a top mass. They considered a tensegrity module that contains six cables and three compressive members, and derived the dynamic model of the tensegrity system using Lagrange's equation with constraints. They performed static and linear dynamic analyses of special tensegrity system. Irfan Baig and Grätsch [22] extensively reviewed the practical use of numerical approaches for linear and non-linear dynamics, however, in their study; they mostly focused on single degree of freedom systems. An elaborate review on the state of the art of dynamic analysis is given by Mirats Tur and Hernandez [23].

Dynamic analysis of structures can be performed using the modal superposition method or the direct time-integration scheme. While linear structures can be analyzed by either method, modal superposition cannot be used for structures that exhibit nonlinearity [24]. This paper provides an extension of Irfan and Grätsch research [22] to multi degrees of freedom systems, such as tensegrity structures. Therefore, the main objective of this work is to highlight the fact that the Newmark approach, as a direct time integration scheme, can be adopted to investigate the behavior of tensegrity structures subjected to dynamic loads. Moreover, the influences of considering damping and slackening condition on dynamic solution of tensegrity structure are studied. In order to describe dynamic behavior of tensegrity structures, a linearized dynamic model is obtained. In order to model accurate tangent stiffness matrix of the tensegrity system, a slackening condition of the cable members is added into the matrix. The slackening occurs when the current length of the cable is shorter than its resting length. In this work, the dynamic external loads are applied on the system such as base excitation or different nodal loadings.

The rest of paper is organized as follows: in Section 2, the analytical formulation used for solving the dynamic response of the structure is described, and fundamental equations of the Newmark method are also derived. In Section 3, three numerical examples are provided; of which two examples are three dimensional grid structures and the others is a two-dimensional cantilever problem. Numerical examples are provided. Based on the findings from the analyses results, Section 4 discusses the conclusions about the use of Newmark time-integration scheme.

2. Analytical formulation

In this work, we assume that the form finding step is already performed, in other words, the feasible nodal coordinates (nodal coordinate), element pre-stressed lengths l_i, and normalized force density coefficients q_i of tensegrity systems are known. The linearized dynamic model can be effectively employed instead of a complete nonlinear dynamic model [4,9,25].

The linearized matrix form of the equation of motion at a pre-stressed configuration along with external forces in global coordinates is as follows:

$$\mathrm{M^g\ddot{u}} + \mathrm{C^g\dot{u}} + \mathrm{K_T^g u = F} \tag{1}$$

where $\mathrm{M^g}$, $\mathrm{C^g}$ and $\mathrm{K_T^g}$ are respectively the mass, damping and tangent stiffness matrices, $\mathrm{\ddot{u}}$, $\mathrm{\dot{u}}$ and u denote the vectors of nodal accelerations, velocity and displacements respectively in global coordinate system. F shows the external dynamic loading.

In theory, the tangent stiffness matrix is composed of two distinctive parts which are material and geometric terms. The material stiffness $\mathrm{K_E^g}$ is often considered for linear analysis of unprestressed structure, and the geometrical stiffness matrix $\mathrm{K_G^g}$ is caused by pre-stresses, [26].

$$\mathrm{K_T^g} = \mathrm{K_E^g} + \mathrm{K_G^g}. \tag{2}$$

In order to model tensegrity structure based on finite element approach, each component of the structure is described by the following mass, damping and stiffness matrices in local coordinates system $\left(\hat{x}\hat{y}\hat{z}\right)$:

$$\mathrm{k_T^1} = \frac{E_i A_i}{\mathrm{l_{0i}}}\begin{bmatrix} \mathrm{I_0} & -\mathrm{I_0} \\ -\mathrm{I_0} & \mathrm{I_0} \end{bmatrix} + \frac{F_i}{\mathrm{l_i}}\begin{bmatrix} \mathrm{I_3} & -\mathrm{I_3} \\ -\mathrm{I_3} & \mathrm{I_3} \end{bmatrix},$$

$$m^1 = \frac{\rho_i A_i \mathrm{l_{0i}}}{6}\begin{bmatrix} \mathbf{2}\mathrm{I_3} & \mathrm{I_3} \\ \mathrm{I_3} & \mathbf{2}\mathrm{I_3} \end{bmatrix},$$

$$c^1 = \beta_1 m^1 + \beta_2 \mathrm{k_T^1},$$

$$\mathrm{I_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \mathrm{I_0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, i = 1, \ldots, b \tag{3}$$

where E_i and A_i denote the elastic modulus and member area respectively. F_i is the pre-stress force, ρ_i is the density of elements, l_{0i} is the rest length, and b is the number of cable and bar elements. β_1 and β_2 denote Rayleigh damping coefficients. It is worth mentioning that, here, the proportional damping is taken into account for modeling the structural damping. The global mass, damping and stiffness matrices are then calculated by combining all individual elements expressed in global coordinate (XYZ). The following modeling assumptions have been made throughout this study:

- The materials are assumed linear elastic and members are prismatic.
- Struts are elements that carry axial tensile or compressive forces.
- Strings are elements that carry only axial tensile forces.
- The tensegrity systems are subjected to external load only at nodes.

The modal analysis of tensegrity system is performed by omitting the damping matrix and the vector of external force in Eq. (1). The eigenvalues and their corresponding eigenvectors of tensegrity structure are calculated by considering a trial solution that exists of the form $\mathrm{u} = \boldsymbol{\varphi}_\mathrm{r} e^{i\omega_\mathrm{r} t}$ where ω_r is the angular frequency, and $\boldsymbol{\varphi}_\mathrm{r}$ is the mass-normalized amplitude vector that has the particular property of $\boldsymbol{\varphi}_\mathrm{r}^\mathrm{T}\mathrm{M}\boldsymbol{\varphi}_\mathrm{r} = \mathrm{I}$. Therefore Eq. (1) leads to:

$$\mathrm{K_T^g}\boldsymbol{\varphi}_\mathrm{r} = \omega_\mathrm{r}^2 \mathrm{M^g}\boldsymbol{\varphi}_\mathrm{r} \tag{4}$$

The spectra decomposition of matrix $(\mathrm{M^g})^{-1}\mathrm{K_T^g}$ is used to determine the eigenvalues and their corresponding eigenvectors of the structural finite element model.

Here, in order to express the behavior of tensegrity structures subject to dynamic loading, a linearized dynamic model near the equilibrium configuration is considered. Accordingly, Newmark method as an implicit direct time integration method is utilized to solve the structural dynamic governing equations of motion for arbitrary external dynamic loading. Implicit methods attempt to satisfy the differential equation at time "$t + \Delta t$" after the solution obtained at time "t" [27]. The basic dynamic equations of the Newmark method are usually formulated as