Journal of Building Engineering 3 (2015) 127–134

Contents lists available at ScienceDirect

Journal of Building Engineering

journal homepage: www.elsevier.com/locate/jobe

Computer-aided architectural designs and associated covariants

Krishnendra Shekhawat *

Department of Mathematics, University of Geneva, Geneva, Switzerland

ARTICLE INFO

Article history:
Received 18 April 2015
Received in revised form
7 July 2015
Accepted 15 July 2015
Available online 17 July 2015

Keywords:
Covariants
Floorplan
Graph
Prototype
Rooms

ABSTRACT

To compare two architectural designs or to characterize them, some numbers are needed. These numbers are said to be *covariants*.

In this paper, we present a software prototype that generates a floor plan design and its adjacency graph for a given set of data, and computes some mathematical covariants associated with the obtained design. In addition, we discuss and demonstrate the usefulness of covariants in comparing the architectural designs and in obtaining a best design among many possible solutions.

1. Introduction

The concept of an *invariant* appeared naturally in geometry in response to the need for the classification of figures. In concrete terms, what distinguishes the following three sub-figures in Fig. 1? Geometrically, the first sub-figure is a *straight line*, the second sub-figure a *rectangle* and the third one a *spiral*. A general line in the same plane intersects in at most 1 and 2 points respectively for the first two figures, while it meets the spiral in infinitely many points. Although the numbers 1, 2 and ∞ do not fully describe these figures, yet they characterize and distinguish them from other geometrical shapes. Interestingly, these numbers remain unchanged if we apply certain geometric transformations, like isometries or scale changes. Thus the *number of intersection points* is an *invariant* with respect to a restricted set of transformations, which can be specified.

If a number or a mathematical object associated with a geometric configuration remains unchanged with respect to a certain group of transformations then this number or mathematical object is said to be an *invariant* (with respect to the specified group of transformations).

For the shapes in Fig. 1, the angles of intersection are certainly not invariant under *rotation*, but we can say how they behave. Therefore, the angles of intersection are *relative invariants* or *covariants*, with respect to *rotation*.

If we know precisely how a number or a mathematical object associated with a geometric figure behaves with respect to a specified set of transformations, we call it a *covariant*.

For a better understanding, consider the rectangular floor plan in Fig. 2A. After rotating it by 90°, as shown in Fig. 2B, its area remains unchanged while its width (dimension measured horizontally) and length (dimension measured vertically) are changed. Hence the area of the rectangular floor plan is an invariant while its width and length are covariants, with respect to the transformation *rotation*.

In the literature, there exist some work where the concepts of invariants and covariants were introduced and studied [1–4]. In this paper, we will explore more about them. Also, we will demonstrate their usefulness in identifying and classifying the different architectural designs, which will present them as a very powerful tool for the architects.

To proceed further, let us consider the plus-shape floor plan, illustrated in Fig. 3, which is generated using the algorithms given in [5]. A plus-shape, as shown in Fig. 4A, can be visualized as a shape made up of 5 rectangles, as illustrated in Fig. 4B. In this way, the plus-shape floor plan in Fig. 3 is generated by adjoining 5 rectangular blocks where each of them is *best connected*[1] and is constructed using the spiral-based algorithm given in [6]. Therefore, we call the plus-shape floor plan described in this paper by spiral-based plus-shape floor plan and represent it by $F_S^p(m)$ (floorplan plus-shape spiral-based) where m is the number of

[1] A rectangular floor plan is best connected if it has $3n-7$ edges in its adjacency graph where n is the number of rooms i.e. there does not exist any other rectangular floor plan with n rooms whose adjacency graph has more than $3n-7$ edges (for details, refer [5]).

* Correspondence address: Present address: CIAUD, Faculdade de Arquitectura, Universidade de Lisboa, Portugal.
E-mail address: krishnendra.iitd@gmail.com

http://dx.doi.org/10.1016/j.jobe.2015.07.005
2352-7102/

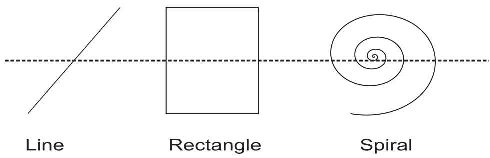

Fig. 1. Understanding the concept of invariants and covariants.

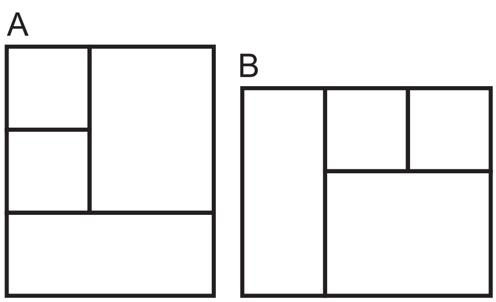

Fig. 2. Invariants and covariants associated with a rectangular floor plan.

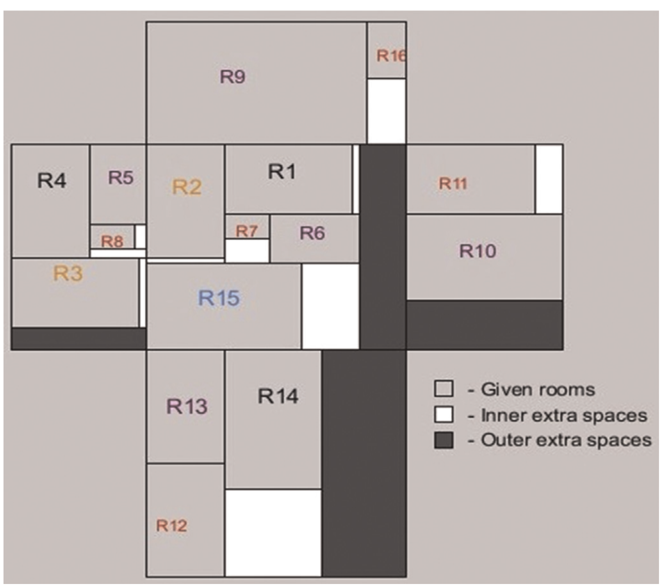

Fig. 3. A computer-generated $F_S^P(16)$.

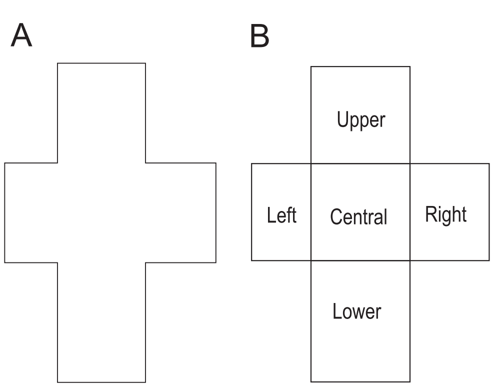

Fig. 4. A plus-shape rectilinear polygon and its division into rectangles.

rooms given by the architects. Also, its graph is represented by $G_S^P(m)$ (graph plus-shape spiral-based). A spiral-based rectangular block (or floor plan) is denoted by F_S^R (floorplan rectangular spiral-based).

If it is required to produce a larger rectangle by arranging different size rectangular pieces (without changing the width and length of rectangular pieces), undoubtedly there would be some empty spaces (or extra spaces) inside the produced rectangle. Therefore, we can see the presence of empty spaces as white rectangles inside the $F_S^P(16)$ in Fig. 3, which we call by inner extra spaces.

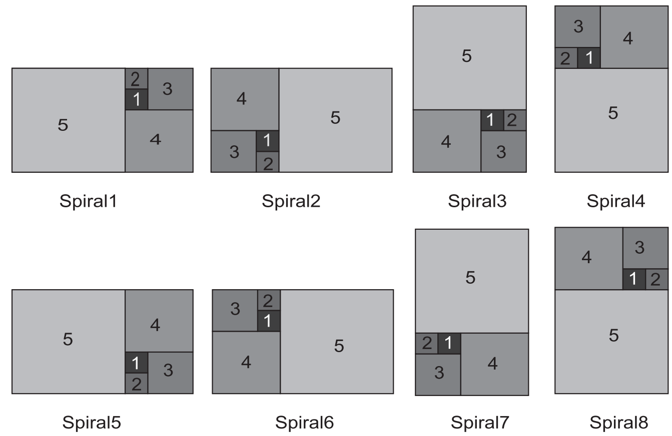

Fig. 5. 8 different F_S^R congruent to each other.

If 5 different size F_S^R are adjoined to generate a F_S^P, again there would be some empty spaces, which we call by outer extra spaces, as shown in Fig. 3.

In [6], it has been illustrated that a F_S^R is congruent to 7 other F_S^R which are best connected and one can be derived from other by four types of mappings i.e. translations, reflections, rotations, and glide reflections (the last being a combination of a translation and a reflection). For better understanding, refer to Fig. 5 with eight congruent F_S^R (with no extra spaces), called by spiral1, spiral2, spiral3, spiral4, spiral5, spiral6, spiral7, and spiral8 F_S^R respectively. As an example, we can see that the $F_S^P(16)$ in Fig. 3 is constructed by considering spiral8, spiral6, spiral4, spiral2 and spiral1 F_S^R for the central, left, upper, right and lower positions respectively (these positions are illustrated in Fig. 4B).

From Fig. 3, we can notice that, for the construction of a F_S^P, five F_S^R are required which are placed at five different positions and each F_S^R can be generated in eight different ways, therefore, for a given set of data, $8^5 = 32,768$ F_S^P can be generated, which is a big number. For example, in Fig. 6, a F_S^P is generated from spiral1, spiral2, spiral3, spiral1 and spiral6 F_S^R for the central, left, upper,

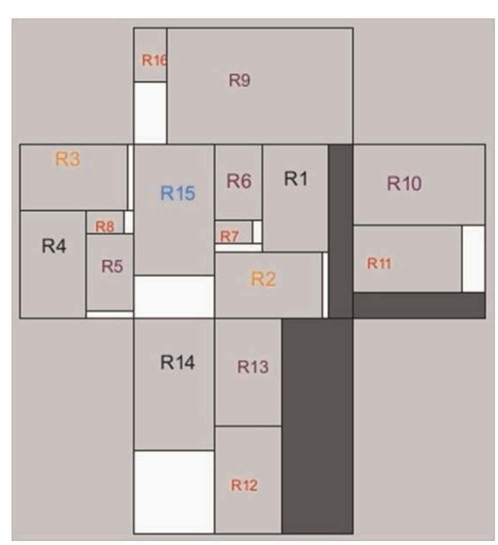

Fig. 6. A computer-generated $F_S^P(16)$.