Available online at www.sciencedirect.com
ScienceDirect
J China Univ Mining & Technol 18 (2008) 0531–0536
JOURNAL OF CHINA UNIVERSITY OF MINING & TECHNOLOGY
www.elsevier.com/locate/jcumt

# Application of remote-sensing-image fusion to the monitoring of mining induced subsidence

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**Abstract:** We discuss remote-sensing-image fusion based on a multi-band wavelet and RGB feature fusion method. The fused data can be used to monitor the dynamic evolution of mining induced subsidence. High resolution panchromatic image data and multi-spectral image data were first decomposed with a multi-ary wavelet method. Then the high frequency components of the high resolution image were fused with the features from the R, G, B bands of the multi-spectral image to form a new high frequency component. Then the newly formed high frequency component and the low frequency component were inversely transformed using a multi-ary wavelet method. Finally, color images were formed from the newly formed R, G, B bands. In our experiment we used images with a resolution of 10 m (SPOT), and TM30 images, of the Huainan mining area. These images were fused with a trinary wavelet method. In addition, we used four indexes—entropy, average gradient, wavelet energy and spectral distortion—to assess the new method. The result indicates that this new method can improve the clarity and resolution of the images and also preserves the information from the original images. Using the fused images for monitoring mining induced subsidence achieves a good effect.
**Key words:** remote sensing image; image mosaic; mining subsidence; multi-band wavelet

## 1 Introduction

Remote sensing data can meet the needs of monitoring a large mining area. These data intuitively and conveniently reflect mining induced subsidence and obtain a panoramic distribution of information covering a continuous space. Remote sensing image fusion is an important part of remote technology applied to monitoring mining induced subsidence. It is a major way of maximizing remote sensing information. Multi-source remote sensing image data obtained of the same target area from several sensors keeps increasing. Fusing a panchromatic image, with high spatial resolution, to a multi-spectral image of low spatial resolution may remedy a data deficiency in a single image. This can enlarge the scope of both data sets in a way that improves interpretation accuracy. In our binary study, we use the remote sensing image fusion technique to monitor mining induced subsidence. We discuss the method of enhancing monitoring accuracy by using the multi-ary wavelet transform model on varying, dynamically evolving, TM and SPOT image fusion data.

## 2 Multi-ary wavelet transform model

### 2.1 Scale analysis of multi-ary wavelets

Remote sensing image fusion can decrease the incompleteness, the multiplicity and the uncertainty existing when interpreting the target objects. So image fusion can greatly improve the effectiveness of feature extraction and classification and target recognition. Wavelet Transform (WT) techniques have the advantages of Fourier analysis while overcoming the contradictions between time and frequency resolution. WT has good local characteristics in both the frequency and space domains. A binary wavelet transform can be performed on images whose resolution ratio is an integral multiple of 2. When this is not the case, we should use a multi-ary wavelet transform[1–2].

Multi-scale analysis is the basic theory of the multi-ary wavelet transform model. Compared to the binary wavelet transform its transform is $M$-ary discrete. Using multi-scale analysis theory we get an orthogonal decomposition in square integrable func-

Received 12 February 2008; accepted 25 July 2008
Project 2003-38 supported by the Geological Investigation Item of Anhui Province
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tion space $L^2(R)$[3].

If $V_{j+1}=W_j^s\oplus V_j\left(1\le s\le M-1\right)$, $j\in Z$, then for every integer $N$ and $M(>0)$, the following formula holds:

$$V_N=W_{N-1}^s\oplus W_{N-2}^s\oplus\cdots\oplus W_{N-M}^s\oplus V_{N-M}\quad(1)$$

So, we have

$$\forall f_N\in V_N,\ g_{N-M}^s\in W_{N-M}^s,$$

and the following formula holds:

$$f_N=g_{N-1}^s+g_{N-2}^s+\cdots+g_{N-M}^s+f_{N-M}\quad(2)$$

The closed subspace column $V_j$ is generated from the orthogonal basis $\left\{\varphi_{j,k}=M^{\frac{j}{2}}\varphi\left(M^jx-k\right)\middle|K\in Z\right\}$, while $W_j$ is generated from

$$\left\{\psi_{j,k}^{(s)}=M^{\frac{j}{2}}\psi_{j,k}^{(s)}\left(M^jx-k\right)\middle|1\le s\le M-1,K\in Z\right\}$$

where $\varphi(x)$ is the scale function and $\left\{\psi^{(s)}\middle|1\le s\le M-1\right\}$ the wavelet function that satisfies these equations:

$$\begin{cases}\varphi(x)=\sum\limits_{k\in Z}h_k\varphi(Mx-k)\\ \psi_s(x)=\sum\limits_{k\in Z}g_k^s\varphi(Mx-k)\end{cases}\quad(3)$$

The filtering equation is then:

$$H(z)=\frac{1}{M}\sum_{k\in z}h_kz^k\quad(4)$$

where $h_k$ is the filtering coefficient and $g_k^s$ is the wavelet coefficient. There is an orthogonality relationship between these:

$$\begin{cases}\sum\limits_{k\in Z}h_k\overline{h}_{k+ml}=\delta_{l,0}\\ \sum\limits_{k\in Z}h_k\overline{g}_{k+ml}^{(s)}=0\\ \sum\limits_{k\in Z}g_k^{(s_1)}\overline{g}_{k+ml}^{(s_2)}=\delta_{s_1,s_2}\delta_{l,0}\end{cases}\quad(5)$$

### 2.2 Multi-ary wavelet analysis

Here we use the multi-ary wavelet to decompose a 2-D energy-limited signal $f^2(x,y)\in L^2(R)$ representing an image generated by remote sensing[3]. First we solve:

$$C_{j+1,k,l}=\sum_m\sum_n\overline{h_{m-Mk}h_{n-Ml}C_{j,m,n}}$$

$$D_{j+1,k,l}^{l,s}=\begin{cases}\sum\limits_m\sum\limits_n\overline{h_{m-Mk}g_{n-Ml}^s}C_{j,m,n}\\ \sum\limits_m\sum\limits_n\overline{g_{m-Mk}^lh_{n-Ml}}C_{j,m,n}\\ \sum\limits_m\sum\limits_n\overline{g_{m-Mk}^lg_{n-Ml}^s}C_{j,m,n}\end{cases}\quad(6)$$

where $j=0,1,2,\cdots$, $\left\{C_{j,k,l}\right\}$ is the low frequency component in the $j$th layer and $\left\{D_{j,k,l}^{l,s}\right\}$ is the high frequency component in the $j$th layer. $M$-ary wavelet decomposition can generate one low frequency component and $M^2-1$ high frequency components.

The reconstructed equation is:

$$C_{j,k,l}=\sum_m\sum_nh_{k-Mm}h_{l-Mn}C_{j+1,m,n}+\sum_{l,s=0,s+l\neq0}^{M-1}\sum_m\sum_ng_{k-Mm}^lg_{l-Mn}^sD_{j+1,m,n}^{l,s}\quad(7)$$

## 3 Image fusion by a multi-ary oriented wavelet transform

### 3.1 Multi-ary wavelet structure

Compared to a binary wavelet the multi-ary wavelet is complex in structure. For $M=2$ there is only one wavelet function, which can be expressed over the integer $\varphi$. For $M>2$ we have $M-1$ wavelet functions: $\left\{\psi_s(x)\middle|1\le s\le M-1\right\}$, which cannot be determined by the scale function $\varphi$ [4–5]. Reference [3] gave the formula for constructing an $M$-ary wavelet scale function filter with a regularity of $k$th order as:

$$H_0(z)=\left[\frac{1+z^{-1}+\cdots+z^{-(M-1)}}{M}\right]^kQ(z)\quad(8)$$

Eq.(8) shows that there is a $Q(z)$ which makes $H_0(z)$ a standard scale filter. For the filter length $N=Mk$, $Q(z)$ is a polynomial of order $k-1$ with respect to $z^{-1}$. Hence $z^{k-1}Q(z)Q\left(z^{-1}\right)$ is the polynomial of order $2(k-1)$ with respect to $z$.

Using this method we can construct the following multi-ary wavelet where $M=3$ and $k=2$:
$h_0=0.19864731$, $h_1=0.31135833$, $h_2=0.43128827$, $h_3=0.13389235$, $h_4=0.025735146$, $h_5=-0.079372413$;