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Element-based effective width for deflection calculation of steel-concrete composite beams

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ABSTRACT

Traditional definition of the effective width of steel-concrete composite beam is based on a certain beam section. Section-based effective width changes from one section to another along the beam span. Therefore, effective width for deflection calculation of composite beam should be evaluated based on an element, rather than a specific section. This paper firstly presents the development of two theoretical models for composite beams. One is shear-lag slip beam model (SSM), which takes into consideration both slip effects and shear-lag effects. The other is slip beam model (SLM), which only considers the interface slip between steel beam and concrete slab. Validation of the theoretical models is performed through comparison of the theoretical predictions with the results obtained from more complex finite element simulations. Based on the theoretical models, an element-based definition of effective width for deflection analysis of composite beams is proposed. Parametric studies are performed to find out the most important parameters influencing effective width. It is demonstrated that the effective width is mostly related to the width of the concrete slab, the span of the beam and the thickness of the floor slab. Simplified design formulas for computing the effective width are proposed. Comparisons between the results of the simplified formulas and the test results indicate the accuracy of the proposed formulas.

1. Introduction

Steel-concrete composite beams have been extensively used in civil and infrastructure engineering in recent years because they combine the advantage of the two construction materials [1–4]. Because shear strain exists in the plane of concrete slab, the longitudinal strain of the portion of the concrete slab away from steel beam lags behind that of the portion near the beam. This phenomenon is called shear-lag effect, which results in a non-uniform strain distribution across the transverse direction of the slab. In the last decades, there has been growing interest in shear-lag effects of T-beam structures and steel-concrete composite beams. Reissner [5] first studied the shear-lag effect of a symmetric rectangular box girder based on the principle of minimum potential energy. Zhu et al. proposed an elaborate element model based on effective width to account for the shear-lag effect of composite decks [6]. Mixed formulations for steel-concrete composite beams with shear lags are useful tools to solve this problem. Dezi et al. developed a model for composite beams with slip and shear-lag effects considering the long-term behavior of the concrete [7]. Sun et al. proposed displacement-based and two-field mixed beam elements for the analysis of composite beams with shear lag and deformable shear connection [8]. Dall'Asta et al. reported a three-field mixed finite element for non-linear analysis of composite beam. Numerical applications are performed using steel-concrete composite cantilever beam elements [9]. Chen derived a closed-form solution on the shear lag based on the energy variational principle for thin-walled box beams. The accuracy of the proposed method is much better than that of conventional method when a function of parabolic curves is used [10]. Fang studied the shear lag effects of tension steel members with welded connections. The resistance factors used in the current codes for predicting the ultimate tensile capacity of single angles and tee sections are examined [11]. Zhang and Lin assumed the deflection induced by shear lag effect as generalized displacement so that shear-lag deformation was separated from flexural deformation. The calculated results agreed well with test results [12].

In order to simplify the analytical evaluation of shear-lag phenomenon, the concept of effective width is introduced in order to utilize conventional beam theory [13–17]. Amadio et al. conducted an experimental study on the evaluation of effective width for steel-concrete composite beams. They proposed a simple modification of the Eurocode 4 for the effective width of composite beams under hogging bending moment [18]. Castro et al. found that the effective width is mainly related to the actual slab width as well as the beam span [19]. Chen and Aref proposed simplified formulas for computing effective width for steel-concrete composite bridge girders subjecting to both positive and negative moment. The proposed formulas were well verified by companion experiments [20–22]. Macorini and Fan conducted analyses on composite beams subjected to long-term loading. The influence of shrinkage and cracking of concrete on the effective width was evaluated [23,24]. Nie et al. presented a new definition of

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effective width for ultimate analysis of the composite beam under sagging moments. Simplified design formulae of the effective width were proposed for the ultimate strength design [25,26]. The concept of effective width also applies to cold formed steel sections [27–30] and angle connections [31–34].

Most of existing studies concerning the definition of the effective width are based on a specific section. However, the section-based effective width changes from one section to another along beam span. Thus, the effective width for calculating the stiffness of the composite beam should be evaluated based on an element, rather than a section. This paper mainly focuses on the effective width for the deflection calculation of composite beams. Theoretical models of the composite beams are developed first considering interface slip and shear-lag effect. Comparisons between the predictions of the theoretical model and those of FE model indicate that the theoretical model can accurately capture the macro behavior and micro response of the composite beam. Then an element-based effective width is defined based on the theoretical model. Next, parametric analyses are conducted to explore the key factors affecting the coefficient of effective width. And simplified design formulas for calculating the coefficient of effective width are proposed.

2. Theoretical model of composite beams

In the analysis of composite beams, it is a common place to determine the section of the beam first by using the notation of effective width to take into account shear-lag effects. In order to make direct comparisons between beams with and without shear-lag effects, two kinds of theoretical models are developed. They are the shear-lag slip beam model (SSM) and the slip beam model (SLM), respectively. For the SSM, the slip effects are considered by introducing interface spring. And the shear-lag effects, which result in non-uniform distribution of strains along the width of the concrete slab, are considered by incorporating shear warping functions. For the SLM, only slip effects are considered. The assumption of plane section is applied to steel beam and concrete slab, respectively, in the SLM. The classical Euler–Bernoulli beam theory is therefore, applicable to the steel beam and the concrete slab, respectively. It is assumed that there is no vertical separation between the steel beam and the concrete slab so that the deflection of the steel beam and that of the concrete slab are identical in both SSM and SLM. The Cartesian coordinate system is used to describe the direction and deformation of the composite frame beams, as shown in Fig. 1. The basic unknowns are the transverse deflection $v(x)$, the longitudinal displacement of the centroid of the steel beam $u_{s0}(x)$, and the longitudinal displacement of the centroid of the concrete slab $u_{c0}(x)$. The slip at the interface $s(x)$ is incorporated to consider the slip effects.

2.1. SSM

In order to model shear lag effects, the longitudinal displacement of concrete slab is expressed as a product of a shape function $\varphi(y)$ and an intensity function $f(x)$ of shear warping displacements as depicted in Fig. 2. The shape function $\varphi(y)$ is an assumed function which depends only on the variable x, so that the shape of shear warping displacements is the same along the slab span while the warping displacements are constant along the depth of the slab. This kinematics assumption was developed by Reissner [5] to model shear lag. It attracts worldwide attention because of its advantage of conciseness and brevity with which the complex effects of shear lag can be expressed with only one additional unknown $f(x)$. The longitudinal displacement of the concrete slab may be expressed as:

$$u_c(x,y,z) = u_{c0}(x) - (z - z_{c0})v' - f(x)\varphi(y) \tag{1}$$

where $\varphi(y)$ is the shape function, $f(x)$ is the shear warping function. If $f(x)$ equals zero, the shear-lag effect will not exist. That is, the strains in the concrete slab are uniform along the transverse direction of the beam. For simplicity, the normal strains in the y and z direction are ignored in the analysis because they are far less than those in the x direction.

The geometry mechanism of SSM is shown in Fig. 3. The geometry equations for SSM are given by:

$$d_c v'(x) = u_{s0} - u_{c0} + s(x) \tag{2}$$

$$\varepsilon_c = u_{c0}'(x) - (z - z_{c0})v'' - f'(x)\varphi(y) \tag{3}$$

$$\gamma_c = -f(x)\varphi'(y) \tag{4}$$

$$\varepsilon_s(x) = u_{so}'(x) - (z - z_{s0})v''(x). \tag{5}$$

In order to determine the shape function $f(x)$, we shall follow these basic rules. The longitudinal strains in the concrete slab reach their maximum and minimum value at $y = 0$ and $y = b$, respectively. The shear strains in the concrete slab follow the same pattern of variation. Thus, the boundary conditions of the shape function may be expressed in mathematical terms:

$$\varphi(0) = 0, \quad \varphi(b) = 1, \quad \varphi'(0) = 1, \quad \varphi'(b) = 0. \tag{6}$$

Here we use the sine function as the shape function because it is concise and satisfies all the boundary conditions in Eq. (6):

$$\varphi(y) = \sin\left(\frac{\pi y}{2b}\right). \tag{7}$$

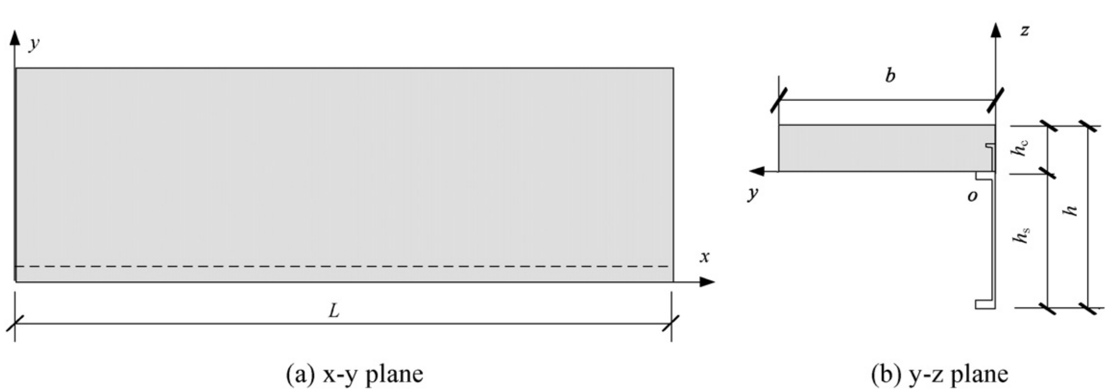

Fig. 1. Notation and coordinate system of the composite beam.