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Second-order plastic-hinge analysis of planar steel frames using corotational beam-column element

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ABSTRACT

A new beam-column element for nonlinear analysis of planar steel frames under static loads is presented in this paper. The second-order effect between axial force and bending moment and the additional axial strain due to the element bending are incorporated in the stiffness matrix formulation by using the approximate seventh-order polynomial function for the deflection solution of the governing differential equations of a beam-column under end axial forces and bending moments in a corotational context. The refined plastic-hinge method is used to model the material nonlinearity to avoid the further division of the beam-columns in modeling the structure. A Matlab computer program is developed based on the combined arc-length and minimum residual displacement methods and its results are proved to be reliable by modeling one or two proposed elements per member in some numerical examples.

1. Introduction

In the nonlinear analysis of steel structures, the beam-column method has been considered as the simple and effective one in modeling the second-order and inelastic effects and its results are verified to be accurate enough for practical design application as studied by Lui and Chen [1], Liew et al. [2], Chan and Chui [3], Thai and Kim [4], Ngo-Huu and Kim [5], etc. However, the use of the accurate stability functions obtained from the closed-form solution of the beam-column under end axial forces and bending moments can lead to some difficulties in derivation of the stiffness matrix formulation, especially in corotational context. Chan and Zhou [6] proposed the approximate fifth-order polynomial displacement function of the beam-column element and formulated the element stiffness matrix considering the second-order effect by principle of stationary total potential energy. The advantage of using this polynomial function is its simplicity in formulation while its accuracy is still maintained as the use of closed-form stability functions.

The corotational method has been widely used due to its efficiency in deriving the formulation of geometrically nonlinear beam-column element for elastic analysis (Nguyen [7], Le et al. [8]) and inelastic analysis (Balling and Lyon [9], Thai and Kim [10], Saritas and Koseoglu [11]). This study proposes a new seventh-order polynomial displacement function for the approximate solution of the governing differential equations to formulate the element stiffness matrix considering the second-order effect following the beam-column theory in corotational context as presented by Balling and Lyon [9]. The bowing effect is integrated in the formulation to consider the change in element length due to the bending of the element. The refined plastic-hinge method is used to simulate the inelastic behavior of the steel material as lumped concept. To solve the system of equilibrium nonlinear equations, the arc-length combined with minimum residual displacement methods are employed due to their robustness in nonlinear analysis application. A computer program is developed using the Matlab programing language to automate the analysis of nonlinear behavior of planar steel frames under static loads. The obtained analysis results are compared to those of existing studies to verify the reliability and effectiveness of the proposed program.

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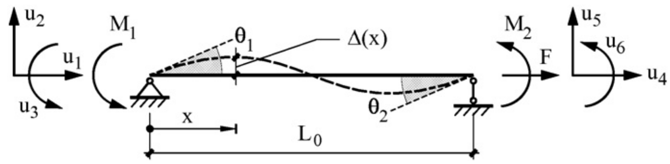

Fig. 1. Simply-supported beam-column element.

2. Formulation

2.1. Stability functions

Consider a simply supported planar beam-column element under end axial force and bending moments as presented in Fig. 1.
The governing differential equations of the element using second-order Euler beam theory are

$$EI\left(\frac{d^4\Delta(x)}{dx^4}\right)-F\left(\frac{d^2\Delta(x)}{dx^2}\right)=0. \tag{1}$$

The closed-form solution to the differential equations leads to following end moment-end rotation relationship (Oran [12])

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}=\frac{EI}{L_0}\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}. \tag{2}$$

For compressive $F<0$

$$\begin{aligned} s_{11}=s_{22}&=\frac{\lambda\sin\lambda-\lambda^2\cos\lambda}{2-2\cos\lambda-\lambda\sin\lambda} \\ s_{12}=s_{21}&=\frac{\lambda^2-\lambda\sin\lambda}{2-2\cos\lambda-\lambda\sin\lambda} \end{aligned} \tag{3}$$

where $\lambda=L_0\sqrt{\frac{|F|}{EI}}$.
For tensile $F>0$

$$\begin{aligned} s_{11}=s_{22}&=\frac{\lambda^2\cosh\lambda-\lambda\sinh\lambda}{2-2\cosh\lambda+\lambda\sinh\lambda} \\ s_{12}=s_{21}&=\frac{\lambda\sinh\lambda-\lambda^2}{2-2\cosh\lambda+\lambda\sinh\lambda}. \end{aligned} \tag{4}$$

For the simplicity in mathematical handling, instead of using the closed-form solution with above-mentioned complicated stability functions, the deflection solution is assumed in following seventh-order polynomial function

$$\Delta(x)=a_7x^7+a_6x^6+a_5x^5+a_4x^4+a_3x^3+a_2x^2+a_1x+a_0. \tag{5}$$

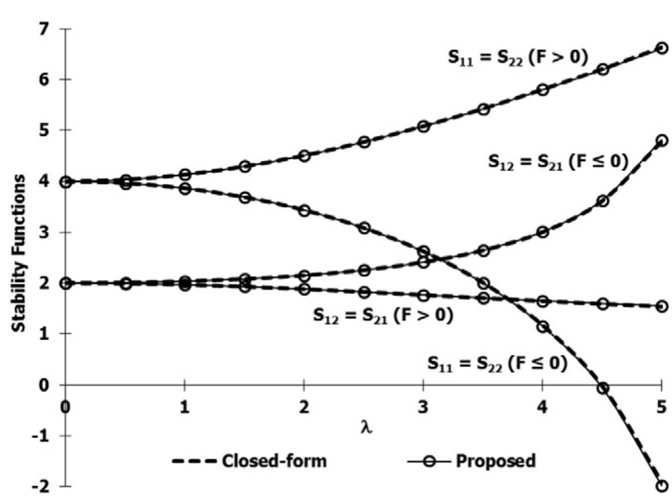

Fig. 2. Comparison of proposed and closed-form stability functions.