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Methods for parameter identification in oscillatory networks and application to cortical and thalamic 600 Hz activity

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Abstract

Directed information transfer in the human brain occurs presumably by oscillations. As of yet, most approaches for the analysis of these oscillations are based on time–frequency or coherence analysis. The present work concerns the modeling of cortical 600 Hz oscillations, localized within the Brodmann Areas 3b and 1 after stimulation of the nervus medianus, by means of coupled differential equations. This approach leads to the so-called parameter identification problem, where based on a given data set, a set of unknown parameters of a system of ordinary differential equations is determined by special optimization procedures. Some suitable algorithms for this task are presented in this paper. Finally an oscillatory network model is optimally fitted to the data taken from ten volunteers.

Keywords: Somatosensory evoked potentials; Somatosensory evoked fields; Parameter identification; Oscillatory networks; 600 Hz activity

1. Introduction

Particularly in chemistry [1], physics [19,31,36,41], biochemistry [30], biology [5], geography [6] and medicine [21] oscillatory network models are utilized for explaining the underlying principles and effects of the observed data. Each system is governed by its own parameter set. In practice, the model is often known, but the actual parameters (at least some of them), e.g. mass or spring constant in a mass spring system, remain unknown. The only possibility to determine these parameters is to fit the model to observed data.

If the fitting procedure is accomplished, the determined parameters should usually give a good estimate of the actual unknown parameters. This general task is called parameter identification; the process involves deriving a model to fit the model by using a variety of numerical tools and a statistical evaluation of the fitted model.

Oscillatory networks (coupled oscillators) are mathematical models for the description of different phenomena in many areas of science and engineering. Particularly, they were applied to the study of non-linear dynamics and synchronization [13,16,27,38,39], in chemistry [22], in pattern recognition and in image processing [2,4,23], and to the modeling of synchronization of rhythmic brain activity [12,37]. Effects like e.g. superposition, synchronization or chaos may be observed in oscillatory networks and depend on the intrinsic mechanism of oscillation and on the nature of the network couplings.

In the present paper, we review the process of parameter identification in combination with oscillatory networks and provide a list of useful tools to accomplish this task. Furthermore, we introduce a two-dimensional forced coupled oscillatory network for electrical 600 Hz

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brain activities in the areas 3b and 1 including forcing thalamic activity, which can be assumed when peripheral nerves are stimulated [10,11,15,33]. The origin and function of these 600 Hz activities are not completely understood yet, but a causal connection seems to be likely. The aim was to derive a fairly uncomplicated model which may describe the spatial–temporal features of these 600 Hz activities. The model may be used to analyze the directed information transfer of the somatosensory system.

2. Materials

Somatosensory evoked potentials and magnetic fields were simultaneously derived from 10 healthy volunteers after an electrical stimulation of the nervus medianus (7000 averages, 5 kHz sampling frequency, 1500 Hz anti-aliasing low pass filter). Additionally, a 3D MRI data set was acquired from each volunteer.

The measurement was performed with 31 magnetic channels (Phillips, Hamburg, Germany) and 32 electrical channels (Neuroscan, El Paso, USA). The stimulation was given by rectangular electrical impulses of duration 0.2 ms. The stimulation frequency was 4 Hz. For each volunteer, the individual sensory and motor thresholds were determined, and the stimulation was applied with a current corresponding to the sum of both thresholds. The stimulated arm was covered during the measurement in order to avoid a cooling. To improve the signal to noise ratio, the registered data were filtered offline by means of an optimal filter (Wiener filter) and were additionally digitally filtered by a third-order Butterworth filter with a band of 450–750 Hz. Afterwards, a singular value decomposition was performed and the noise proportion was eliminated. Electrical as well as magnetic data were used simultaneously for a source reconstruction.

The 3D MRI data of the 10 volunteers were used for the computation of an individual head model consisting of the three components skin, cranium and brain. The individual head models were used for the subsequent source localization [14].

The locations of the Brodmann Areas 3b and 1 were localized by fitting a 2-dipol-model with the help of the Nelder–Mead Simplex algorithm. The activity of the thalamus could not be localized. The activation time courses of the 600 Hz activity formed the observation data for the fitting of the oscillatory network models.

3. Methods

3.1. Oscillatory networks

Generally, an oscillatory network is described by a set of coupled oscillators. In order to introduce an oscillatory network, firstly the notion of an oscillator is formally given in this section. Furthermore, a possible way for coupling different oscillators is described.

There are many known different oscillator types, which are commonly formalized mathematically by ordinary differential equations of first or second-order, respectively. Without loss of generality, we would like to focus on a formal description of an oscillator by the second-order differential equation. That is, the temporal behavior of an oscillator is defined by a differential equation

$$\ddot{x} = f(x, \dot{x}, t), \quad x(0) = x_0, \quad \dot{x}(0) = \chi_0, \tag{1}$$

where $x(t)$ denotes the state of the oscillator at time t, and $\dot{x}(t)$ stands for its velocity. The function f describes the dynamic of the oscillator. At time zero, the oscillator has the starting state x_0 and the starting velocity χ_0. When the right-hand side of Eq. (1) depends linearly on the state x and the velocity, an oscillator is termed a linear oscillator. Otherwise, it is called non-linear. The most simple and best known linear oscillator is the damped harmonic oscillator given by

$$\ddot{x} = -(2\pi\varpi)^2 x - \mu\dot{x}, \quad x(0) = x_0, \quad \dot{x}(0) = \chi_0. \tag{2}$$

In this notation, ϖ defines the frequency, and the starting values x_0 and χ_0 give the amplitude and phase of oscillation implicitly. The non-negative parameter $\mu \geqslant 0$ is called a damping parameter and describes how fast a possible amplitude depression occurs. A vanishing damping parameter results in a harmonic oscillation. Adding an additional time-variant "force" to the right-hand side of Eq. (2), a so called forced oscillator results. Formally, such an oscillator is defined by

$$\ddot{x} = -(2\pi\varpi)^2 x - \mu\dot{x} + g(t), \quad x(0) = x_0, \quad \dot{x}(0) = \chi_0 \tag{3}$$

with a continuous function g.

Let y be an arbitrary second oscillator. Its state y as well as its velocity $\dot{y}$ are continuous functions in time. Consequently, these two functions may serve as a time-variant force for the oscillator x. On the other hand, the oscillator x may be used to force y. In this way, it is easy to formulate an oscillatory network of two linear oscillators e.g. by

$$\begin{aligned} \ddot{x} &= -(2\pi\varpi_x)^2 x - \mu_x\dot{x} + \varepsilon_{xy} y, \quad x(0) = x_0, \quad \dot{x}(0) = \chi_0, \\ \ddot{y} &= -(2\pi\varpi_y)^2 y - \mu_y\dot{y} + \varepsilon_{yx} x, \quad y(0) = y_0, \quad \dot{y}(0) = \eta_0. \end{aligned} \tag{4}$$

Instead of coupling the oscillators by means of the states, it is also possible to couple by means of velocity. To decide which type of coupling should be chosen for a certain problem depends on content related aspects which need to be taken into consideration. Furthermore, the coupling does not depend on the linear type of oscillators. The same construction is valid for arbitrary oscillators. Moreover, it is not necessary to choose the same type of network oscillators. Here for the sake of