Journal of Thermal Biology 49-50 (2015) 98–105

Contents lists available at ScienceDirect

Journal of Thermal Biology

journal homepage: www.elsevier.com/locate/jtherbio

A numerical study on dual-phase-lag model of bio-heat transfer during hyperthermia treatment

P. Kumar [a,*], Dinesh Kumar [b], K.N. Rai [a,b]

[a] Department of Mathematical Sciences, IIT-BHU, Varanasi, India
[b] DST-CIMS, Faculty of Science, BHU, Varanasi, India

ARTICLE INFO

Article history:
Received 26 November 2014
Received in revised form
13 February 2015
Accepted 13 February 2015
Available online 17 February 2015

Keywords:
DPL bio-heat conduction model
Finite element wavelet Galerkin method
Boundary condition
Gaussian distribution
Hyperthermia

ABSTRACT

The success of hyperthermia in the treatment of cancer depends on the precise prediction and control of temperature. It was absolutely a necessity for hyperthermia treatment planning to understand the temperature distribution within living biological tissues. In this paper, dual-phase-lag model of bio-heat transfer has been studied using Gaussian distribution source term under most generalized boundary condition during hyperthermia treatment. An approximate analytical solution of the present problem has been done by Finite element wavelet Galerkin method which uses Legendre wavelet as a basis function. Multi-resolution analysis of Legendre wavelet in the present case localizes small scale variations of solution and fast switching of functional bases. The whole analysis is presented in dimensionless form. The dual-phase-lag model of bio-heat transfer has compared with Pennes and Thermal wave model of bio-heat transfer and it has been found that large differences in the temperature at the hyperthermia position and time to achieve the hyperthermia temperature exist, when we increase the value of τ_T. Particular cases when surface subjected to boundary condition of 1st, 2nd and 3rd kind are discussed in detail. The use of dual-phase-lag model of bio-heat transfer and finite element wavelet Galerkin method as a solution method helps in precise prediction of temperature. Gaussian distribution source term helps in control of temperature during hyperthermia treatment. So, it makes this study more useful for clinical applications.

1. Introduction

Cancer is a life threatening disease. It is leading cause of death in developed countries and second leading cause of death in developing countries (Global Cancer Facts and Figs., 2011). Cancer is a group of disease characterized by unlimited growth and spread of abnormal cells. It is caused by both internal (hormones, inherited mutations) and external (Tobacco, infectious disease, etc.) factors. The development of most cancers requires multiple steps that occur over many years, so their early detection and treatment are possible. Hyperthermia is one of the most commonly used thermal therapies for cancer treatment. It is an adjuvant therapy means it is used along with some other treatment modalities like chemotherapy, radiotherapy and surgery to enhance the effectiveness of the treatment. In hyperthermia treatment, the temperature at the site of cancerous cell (tumor) is increased by using some external means (ultrasound, radio-frequency, micro-waves, infrared radiation, magnetically excitable thermoseeds, tube with hot water, etc., Xu et al., 2009) resulting in changing the physiology of diseased cells which leads to apoptosis (cell death). Depending on the degree of temperature raise, it may be classified into thermal ablation ($46\,^{\circ}\mathrm{C} < T < 56\,^{\circ}\mathrm{C}$), moderate hyperthermia ($41\,^{\circ}\mathrm{C} < T < 46\,^{\circ}\mathrm{C}$) and diathermia ($T < 41\,^{\circ}\mathrm{C}$). Further, on the basis of location of the disease, it is classified into local, regional and whole body hyperthermia. Local hyperthermia involves subjection of heat only to a small area such as tumor. Regional hyperthermia involves larger areas such as whole tissue and organ. Whole body hyperthermia is applied to treat metastatic cancerous cells when it spreads throughout the body (Kumar and Challa, 2011). For the success of any type of hyperthermia treatment precise prediction and control of temperature are always needed (Das et al., 2013; Gupta et al., 2013). There are a number of bio-heat transfer equations (Bhowmik et al., 2013) for living biological tissues given by Pennes (1948), Weinbaum and Jiji (1985), Nakayama and Kuwahara (2008) and many others. But due to simplicity Pennes bio-heat transfer equation is used most commonly for interpretation of thermal data. The conduction term in the Pennes bio-heat transfer equation is based on classical Fourier's law

$$q(r, t) = -k\nabla T(r, t). \tag{1}$$

* Corresponding author.
E-mail addresses: pkumar.rs.apm12@itbhu.ac.in (P. Kumar), dineshaukumar@gmail.com (D. Kumar), knrai.apm@itbhu.ac.in (K.N. Rai).

http://dx.doi.org/10.1016/j.jtherbio.2015.02.008
0306-4565/

Nomenclature

c	thermal wave propagation speed, m/s
c_b	specific heat of blood, J/kg K
k	thermal conductivity of tissue, W/m K
L	length of tissue, m
q_m	metabolic heat generation, W/m^3
q_r	spatial heating source, W/m^3
t	time, s
T	temperature of tissue, K
T_b	arterial temperature, K
W_b	perfusion rate of blood, m^3/s/m^3
q	heat flux, W/m^2
y	spatial coordinate, m
τ_q	phase lag of heat flux , s
τ_T	phase lag of temperature gradient, s

Dimensionless variable and similarity criteria

x	dimensionless space coordinate
F_o	Fourier number or dimensionless time
F_{oq}	dimensionless phase lag of heat flux
F_{oT}	dimensionless phase lag of temperature gradient
θ	dimensionless local tissue temperature
θ_b	dimensionless arterial blood temperature
θ_w	dimensionless surface temperature
θ_f	dimensionless ambient temperature
P_f	dimensionless blood perfusion coefficient
P_r	dimensionless external heat source coefficient
P_m	dimensionless metabolic heat source coefficient
K_i	Kirchhoff number
B_i	Biot number
a	dilation parameter
x^*	location of tumor in tissue

It assumes that the heat flux vector $q(r, t)$ and temperature gradient $\nabla T(r, t)$ appear at the same instant of time i.e. thermal signal propagates with infinite speed. It means that any thermal disturbance produced at a certain location will be felt throughout the medium at that instant. In fact, heat is always found to propagate with a finite speed within living biological tissues as they have highly non-homogeneous inner structure. To solve the paradox occurred in the Pennes bio-heat equation, thermal wave model of bio-heat transfer has been proposed based on single-phase-lagging (SPL) (Cattaneo, 1958; Vernotte, 1958) constitutive relation

$$q(r, t + \tau_q) = - k\nabla T(r, t), \tag{2}$$

where a relaxation time τ_q has added to capture the micro-scale response in time. The relaxation time τ_q actually represents the time needed to establish the heat flux when a temperature gradient is suddenly imposed. Although the thermal wave model of bio-heat transfer taken into account of micro-scale responses in time, it does not capture the micro-scale responses in space. Due to this it produces some unusual thermal behavior. In order to consider the effect of micro-structural interactions along with the fast transient effects, a phase lag for temperature gradient τ_T has introduced in SPL constitutive relation

$$q(r, t + \tau_q) = - k\nabla T(r, t + \tau_T). \tag{3}$$

According to this relation, the temperature gradient at a point r at time $t + \tau_T$ corresponds to the heat flux at r at time $t + \tau_q$. The corresponding model is called the DPL model of bio-heat transfer (Tzou, 1996). Recently, Rukolaine (2014) has explained some unphysical effects of DPL model of heat conduction.

Since Pennes bio-heat equation is a modified form of energy equation, so different numerical methods (Wang et al., 2008) are available in literature for solving them like finite difference method (Shen et al., 2005; Yuan, 2009; Pletcher, 2009), finite difference-decomposition method (Gupta et al., 2013), homotopy perturbation method (Gupta et al., 2010) and finite volume method (Versteeg and Malalasekera, 1995). Shih et al. (2007) solved the Pennes bio-heat transfer equation with sinusoidal surface heat flux on the skin as semi-infinite domain. Ahmadikia et al. (2012) did analytic solution of both parabolic and hyperbolic bio-heat transfer equation. Liu and Chen (2009, 2010) solved the DPL bio-heat model using hybrid numerical scheme.

In the present study, we have to obtain the solution of the DPL model of bio-heat transfer equation under most generalized boundary conditions. Discretizing in space co-ordinate,the problem is converted in to a system of O.D.E's with initial conditions. This system of O.D.E's in unknown variables has been solved by Wavelet Galerkin approach. This reduces our problem into Sylvester equation. Solution of this Sylvester equation gives dimensionless temperature. The analytical solution using Laplace transform technique and approximate analytical solution obtained using FEWGM show good agreement. This proves the rationality and reliability of our solution scheme.

2. Mathematical formulation

During hyperthermia treatment, the body tissue which is initially at a constant temperature $T_0 = 37\ ^\circ$C is heated by some external heat source. In order to consider the micro-scale responses in both time and space DPL constitutive relation is used to derive the DPL model of bio-heat transfer

$$q(y, t + \tau_q) = - k\nabla T(y, t + \tau_T), \tag{4}$$

where q is the heat flux, T is the temperature, ∇T is the temperature gradient, k is the thermal conductivity, τ_q is the phase lag of heat flux and τ_T is the phase lag of temperature gradient. τ_q has been introduced to take account of fast transient effects which induces the behavior of thermal wave whereas τ_T represent micro-structural interactions. The heat flux precedes the temperature gradient for $\tau_q < \tau_T$ and temperature gradient precedes the heat flux for $\tau_T < \tau_q$.

In a local energy balance, the one-dimensional energy equation of the present problem is

$$\rho c\frac{\partial T}{\partial t} = - \frac{\partial q}{\partial y} + \omega_b c_b(T_b - T) + q_m + q_r, \tag{5}$$

where ρ and c denote density and specific heat of tissue, respectively, ω_b, c_b, and T_b is the perfusion rate, specific heat and arterial temperature of blood, respectively. q_m is the metabolic heat generation (Rai and Rai, 1999) and q_r is the Gaussian distribution heating source term (Liu, 2011).

Now eliminating q from Eqs. (4) and (5) gives rise to

$$\rho c\tau_q\frac{\partial^2 T}{\partial t^2} + (\rho c + \omega_b c_b\tau_q)\frac{\partial T}{\partial t} + \omega_b c_b T = k\left(\nabla^2 T + \tau_T\frac{\partial(\nabla^2 T)}{\partial t}\right) + (\omega_b c_b T_b + q_m + q_r), \tag{6}$$