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Graded meshes in bio-thermal problems with transmission-line modeling method

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ABSTRACT

In this study, the transmission-line modeling (TLM) applied to bio-thermal problems was improved by incorporating several novel computational techniques, which include application of graded meshes which resulted in 9 times faster in computational time and uses only a fraction (16%) of the computational resources used by regular meshes in analyzing heat flow through heterogeneous media. Graded meshes, unlike regular meshes, allow heat sources to be modeled in all segments of the mesh. A new boundary condition that considers thermal properties and thus resulting in a more realistic modeling of complex problems is introduced. Also, a new way of calculating an error parameter is introduced. The calculated temperatures between nodes were compared against the results obtained from the literature and agreed within less than 1% difference. It is reasonable, therefore, to conclude that the improved TLM model described herein has great potential in heat transfer of biological systems.

1. Introduction

Temperature propagation in biological tissues is a complicated physiological process that includes blood circulation, sweating, metabolic heat generation, and heat dissipation through the hair coat (Xu et al., 2009; Xu and Lu, 2011). Understanding heat flow in biological tissues has implications on thermal medical treatment such as hyperthermia and thermal ablation (Comas et al., 2007; Milan and Carvalho, 2013; Tungjitkusolmun et al., 2001), and thermal comfort of humans (Alahmer et al., 2011; Cheng and Niu, 2011; Foda et al., 2011) and livestock (Maia et al., 2008; da Silva and Maia, 2013; Gebremedhin et al., 1997; Jiang et al., 2005).

Pennes (1948) proposed a simple and powerful mathematical model of thermal propagation in a biological tissue. Pennes' model is referred to as the bio-heat equation (BHE). The resolution of the BHE equation is limited to a one-dimensional system with favorable geometry (Deng and Liu, 2002; Fan and Wang, 2011) and requires numerical models to solve complex geometries (Berg et al., 2011; Chang, 2003).

Transmission-line modeling (TLM) is a numerical method for solving the bio-heat equation (Amri et al., 2011; Bellia et al., 2008a, 2008b). Using the TLM method, it is not easy to have different space discretization in the mesh (de Cogan, 1998; de Cogan et al., 2006; Sadiku, 2009), and cannot include heat source at any part of the mesh (Bellil and Bennaoum, 2013; Bellil et al., 2013). To model a complex geometry, it would be necessary to discretize the geometry into smaller spaces and, consequently, it would require unnecessary refinement, which means more computational time. In addition, modeling radiation and convection heat transfer needs to be done at different location of the mesh but these heat transfer can only be modeled at the boundary.

1.1. Objectives

To improve the transmission-line modeling procedure, the following specific objectives were studied:

(1) Development of graded meshes within the TLM model that enable heat sources to be modeled in all segments of the meshes, and that provide better approximation of complex geometries and reduce memory space and computational time requirements.

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(2) Formulation of a new boundary condition that considers thermal parameters that enables more realistic modeling of complex geometries.
(3) Development of a method for calculating temperature between nodes.
(4) Validation of the improved model by comparing the results predicted against the results obtained from the literature and also against the results obtained using regular mesh in TLM.

2. Procedure

To model a physical system using TLM, the domain must be discretized in time and space (Hoefer, 1989). The temporal discretization is performed by velocity propagation of the voltage pulse in the transmission line, where the time step (Δt) represents each step in the time discretization (Sadiku, 2009). The spatial discretization (Δl) perform an ensemble of transmission lines and circuit elements known as node (Rao, 1999). For example, Fig. 1(a) shows a regular mesh of 10×10 nodes and Fig. 1(b) shows a graded mesh.

The voltage pulse in a transmission line is characterized by the incident and reflected pulses (Christopoulos, 2006). When the incident pulse is reflected at the same time step, the process is known as *scattering*. *Connection* is the process when the incident pulse in the next time step is obtained from the reflected pulse in the actual time step. These processes are essential to ensure that voltage pulses travel between nodes and to the next time step (Weiner, 2010).

In the classical TLM method, temperature is calculated at the nodes (de Cogan, 1998). In this study, a new method of calculating temperature between nodes is developed. This procedure provides more temperature information within the mesh without increasing memory space and computational time.

To use TLM method to simulate Pennes equation, it is necessary to find a node that has the same set of equations as those proposed by Pennes (Minkowycz and Sparrow, 2009), and the set of equations are expressed as

$$-\nabla q'' = \rho c \frac{\partial T}{\partial t} + \omega_b \rho_b c_b T - (\omega_b \rho_b c_b T_b + Q_m) \tag{1}$$

$$q''_x = -k\frac{\partial T}{\partial x}; \quad q''_y = -k\frac{\partial T}{\partial y}; \quad q''_z = -k\frac{\partial T}{\partial z} \tag{2}$$

where ρ (kg/m^3)=density, c (J/(kg °C))=specific heat, T (°C)= temperature, k (W/(m °C))=thermal conductivity of the tissue, ω_b (s^{-1})=blood perfusion of the tissue, ρ_b (kg/m^3)=density of blood, c_b (J/(kg °C))=specific heat of blood, T_b (°C)=temperature of blood, Q_m (W/m^3)=rate of internal metabolic heat generation, and q'' (W/m²)=heat flux.

Considering the geometry, a three-dimensional model can be used to solve the problem but would require a large memory space and computational time. In some situations, the existence of symmetry may reduce the problem to a two- or even one-dimensional problem. For Eqs. (1) and (2), a one-, two- and three-dimensional TLM models will be demonstrated in Sections 2.1, 2.2 and 2.3, respectively. In Section 2.4, the formulation to calculate temperature between nodes, and in Section 2.5, the models used to validate the TLM method will be given.

2.1. TLM node to model the BHE in one dimension

2.1.1. Isomorphism

The proposed TLM node to model Eqs. (1) and (2) in one-dimension is shown in Fig. 2(a). The numbers in the figure represent the ports that are in contact with the adjacent nodes. The voltages are $V(x-\Delta x/2)$ and $V(x+\Delta x/2)$ in ports 1 and 2, respectively, and $V(x)$ represents the voltage at the center of the node. The spatial distribution of nodes and the connection between ports for adjacent nodes is shown in Fig. 2(b).

Using Kirchhoff's law (Alexander and Sadiku, 2012), the expression for current is

$$-\frac{\partial I_x}{\partial x} = C_{dx}\frac{\partial V}{\partial t} + \frac{G}{\Delta x}V - \frac{I_F}{\Delta x} \tag{3}$$

where

$$V \equiv T;\; I_x \equiv q''_x;\; C_{dx} \equiv \rho c;\; G \equiv \omega_b \rho_b c_b \Delta x;\; I_F \equiv (\omega_b \rho_b c_b T_b + Q_m)\Delta x \tag{4}$$

Again, using Kirchhoff's law in port 2, the expression for voltage is

$$\frac{\Delta x}{R_x}\frac{\partial V}{\partial x} = -I_x - \frac{L_x}{R_x}\frac{\partial I_x}{\partial t} \tag{5}$$

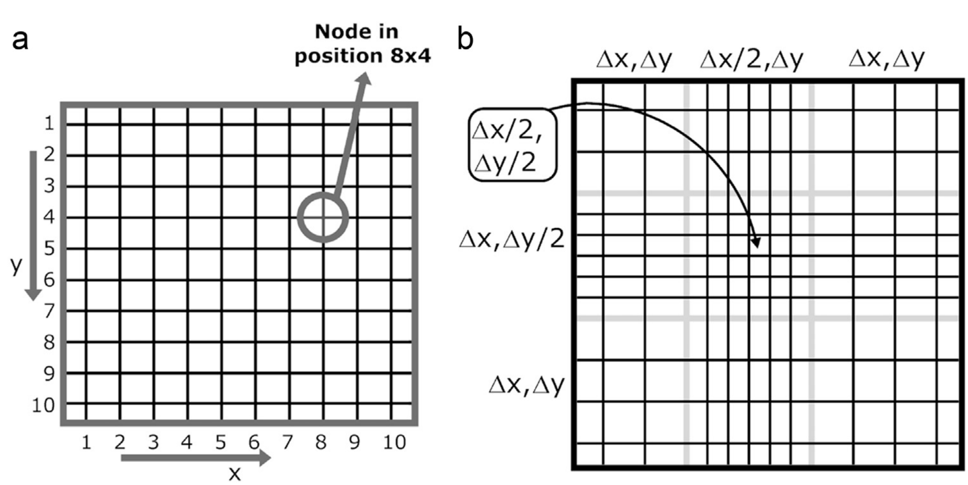

Fig. 1. Two-dimensional TLM (a) regular, and (b) graded mesh composed of 10×10 nodes. Each node is represented by a +. In the graded mesh, the thicker lines represent the change in space discretization.