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# The continuous strength method for the design of circular hollow sections

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ABSTRACT

Circular hollow sections (CHS) are widely used in a range of structural engineering applications. Their design is covered by all major design codes, which currently use elastic, perfectly-plastic material models and cross-section classification to determine cross-section compressive and flexural resistances. Experimental data for stocky sections show that this can result in overly conservative estimates of cross-section capacity. The continuous strength method (CSM) has been developed to reflect better the observed behaviour of structural sections of different metallic materials. The method is deformation based and allows for the rational exploitation of strain hardening. In this paper, the CSM is extended to cover the design of non-slender and slender structural steel, stainless steel and aluminium CHS, underpinned by and validated against 342 stub column and bending test results. Comparisons with the test results show that, overall, the CSM on average offers more accurate and less scattered predictions of axial and flexural capacities than existing design methods.


## 1. Introduction

Circular hollow sections (CHS) have been manufactured and used in structures since the early 1800s as columns, beams, tension members and truss elements [1]. They have become increasingly attractive to designers due to their aesthetic appearance and their benefits over open sections such as superior torsional resistance, bi-axial bending resistance, reduced drag and loading in a fluid, ability to be filled with concrete to form a composite section and their reduced maintenance requirements with a smaller external area exposed to corrosive environments [1]. CHS are primarily thin-walled structural elements, and therefore local buckling, whether prior or subsequent to material yielding, is a primary consideration in their design.

### 1.1. Traditional CHS design methods

Current design codes use the concept of cross-section classification to separate circular hollow sections into discrete classes depending upon their susceptibility to local buckling. Four classes of cross-section are considered in EN 1993-1-1 [2] and BS 5950-1 [3] for structural steelwork, EN 1993-1-4 [4] for stainless steel and EN 1999-1-1 [5] for aluminium. In bending, class 1 cross-sections can reach and maintain their full plastic moment capacity $M_{pl}$ with suitable rotation capacity for plastic design. Class 2 cross-sections are also capable of reaching their full plastic moment capacity but with limited rotation capacity. There is no equivalent to class 2 cross-sections in the AISC 360 [6] and AS 4100 [7] structural steel codes. Class 3 cross-sections are unable to reach their plastic moment capacity due to local buckling and their bending capacity is limited to the elastic moment capacity $M_{el}$. Class 4 cross-sections experience local buckling before reaching their elastic moment capacity, and are typically referred to as slender. In terms of axial resistance, the class 3 limit separates the non-slender cross-sections that are fully effective in compression (i.e. classes 1–3) from those that fail by local buckling before reaching their yield load (i.e. class 4). These traditional design methods also limit the maximum stress in the cross-section to the yield strength $f_y$, neglecting the beneficial effects of strain hardening in metallic materials. Experimental results have shown that cross-section classification and limiting the maximum stress to the yield stress can be overly conservative in estimating the resistance of stocky (classes 1–3, non-slender) cross-sections [8,9]. It is therefore apparent that there are structural efficiency improvements to be sought over existing design methods for CHS.

### 1.2. The continuous strength method

The continuous strength method (CSM) has been developed in recent years to reflect better the observed characteristics of metallic structural elements. Cross-section classification is replaced with a continuous relationship between cross-section slenderness and deformation capacity (referred to in Section 2.5 as the base curve), reflecting the continuous nature of cross-section capacity varying with local slenderness. A strain hardening material model is also adopted, representing the behaviour

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seen in material tests, with an increase in strength above the yield strength under plastic deformation.

The CSM has previously been developed for structural steel [8–11], stainless steel [12] and aluminium [13] plated cross-sections, such as I-sections, square hollow sections (SHS) and rectangular hollow sections (RHS) in compression and bending, and also under combined bending [14]. The previous work has shown that the CSM predicts enhanced capacities over existing methods; for example, in the case of stainless steel, average enhancements in compressive and bending resistances of 12% and 19% respectively were found [12]. The natural progression is to extend the application of the CSM to circular hollow sections, which is the focus of the present paper that builds upon prior work [15], and the development process is described herein.

## 2. Extension of the CSM to CHS design

The process of extending the CSM to cover the design of CHS requires: i) the identification of the yield slenderness limit (i.e. the local slenderness limit below which significant benefit from strain hardening can be derived for non-slender cross-sections); ii) the formulation of the CSM non-slender and slender base curves defining the relationship between cross-section slenderness and deformation capacity; iii) the selection of appropriate material models; and iv) the derivation of resistance functions.

### 2.1. Cross-section slenderness

The local cross-section slenderness $\overline{\lambda}_c$ is defined in a non-dimensional form by Eq. (1),

$$\overline{\lambda}_c = \sqrt{\frac{f_y}{\sigma_{cr}}} \tag{1}$$

where $f_y$ is the material yield strength and $\sigma_{cr}$ is the elastic critical buckling stress, which for a CHS in compression is calculated using Eq. (2),

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{2t}{D} \tag{2}$$

where $E$ is the Young's modulus, $\nu$ is the Poisson's ratio, $D$ is the outer diameter and $t$ is the wall thickness of the CHS.

Timeshenko [16] suggested that the local buckling stress in bending can be taken as 1.4 times that in compression based on experimental results [17], which effectively makes a cross-section in bending more stocky than the same cross-section in compression. Gerard and Becker [18] proposed a factor of 1.3 based upon the findings of Flügge [19]. However more recent analytical work [20–22] has showed that the maximum critical stress in bending is equal to the critical compressive stress for practical cylinder lengths. Differences also exist between international design codes in their treatment of compression and bending for CHS. Gardner et al. [23] noted that EN 1993-1-1 [2] and EN 1999-1-1 [5] utilise the same class 3 limits for both compression and bending, in contrast to BS 5950-1 [3], EN 1993-1-4 [4], AISC 360 [6] and AS 4100 [7] where different limits are used. Utilising different slenderness limits in compression and bending is equivalent to applying a factor to the local buckling stress. Given the findings of the more recent research [20–22] and the conservative nature of the choice, the elastic critical buckling stress in bending will be taken to be the same as that in compression (see Eq. (2)) in the present study and within the CSM.

### 2.2. CHS experimental database

A dataset of 342 experimental results on CHS in compression or bending has been collated from the literature. The dataset includes stub column test results for hot-finished structural steel [24–28], very high strength structural steel [29,30], cold-formed structural steel [26,27,29,31–48], austenitic stainless steel [49–58], duplex stainless steel [59,52,60], ferritic stainless steel [61] and aluminium [62,63], and four-point bending test results for hot-finished structural steel [64–67], very high strength structural steel [68], cold-formed structural steel [35,64,67,69–74], austenitic stainless steel [49,75,76], duplex stainless steel [75] and aluminium [77,78]. Note that the very high strength structural steel had a typical yield stress $f_y$ around 1300 MPa [30]. The number of experimental results used in the definition and assessment of the various aspects of the CSM for CHS, i) the yield slenderness limit (see Section 2.3); ii) the base curve for non-slender sections (see Section 2.5); iii) the base curve for slender sections (see Section 2.5); and iv) the assessment of the capacity predictions (see Section 4) are shown in Tables 1 and 2 for compression and bending respectively. The number of specimens used in the different stages of the extension of the CSM to CHS sometimes varies since not all required parameters were reported in the literature.

### 2.3. Yield slenderness limit

The limiting local slenderness that delineates the transition between slender and non-slender cross-sections needs to be defined. Above this limit there is no significant benefit from strain hardening with the cross-section buckling locally below the yield load in compression or elastic moment in bending. This limit is identified by plotting the ultimate capacity of the stub columns normalised by their yield load ($N_u/N_y$) against cross-section slenderness $\overline{\lambda}_c$, defined by Eq. (1), as shown in Fig. 1. A linear regression fit can then identify the limiting local slenderness where the ultimate axial load equals the yield load, which from Fig. 1 is $\overline{\lambda}_c \approx 0.40$. The class 3 limits from current design codes are also plotted in Fig. 1, and it can be seen that the identified limiting local slenderness is compatible with the class 3 limit for aluminium given in EN 1999-1-1 [5]; however it is above the existing structural steel and stainless steel class 3 limits. There is also some scatter in the stub column dataset. Consequently, a lower value of $\overline{\lambda}_c = 0.3$ for the yield slenderness limit is proposed as this represents approximately a

**Table 1**
Number of CHS stub column test results used in the development of the various aspects of the CSM.

| Material | Yield slenderness limit | Base curve | | CSM and code capacity predictions | |
|---|---|---|---|---|---|
| | | Non-slender sections | Slender sections | Non-slender sections | Slender sections |
| Hot-finished structural steel | 11 | 8 | – | 9 | – |
| Very high strength structural steel | 20 | 1 | 14 | 1 | 19 |
| Cold-formed structural steel | 131 | 48 | 50 | 44 | 52 |
| Stainless steel (total) | 76 | 31 | 26 | 39 | 35 |
| Austenitic stainless steel | 48 | 16 | 13 | 24 | 22 |
| Duplex stainless steel | 21 | 10 | 11 | 10 | 11 |
| Ferritic stainless steel | 7 | 5 | 2 | 5 | 2 |
| Aluminium | 15 | 7 | 7 | 7 | 8 |