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Direct analysis by an arbitrarily-located-plastic-hinge element — Part 1: Planar analysis

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ABSTRACT

This paper, as the first part of the two companion papers, proposes a new and curved beam-column element with arbitrarily-located plastic hinge for second-order inelastic or direct analysis of planar steel frames. Unlike the conventional plastic hinge element which requires two and more elements to capture the locations of plastic hinge along member length causing much inconvenience to model member initial imperfection, one proposed element per member is sufficient for an accurate analysis. The proposed element explicitly models the member initial imperfection allowing for member P-δ effect directly. The plastic hinge is simulated as a gradually softening spring and the corresponding formulae are presented. The additional degrees of freedom in the element are condensed so that the element can be easily incorporated into existing computer programs with dramatic improvement on numerical efficiency and accuracy. Finally, several examples are presented for verifying the accuracy and validity of the proposed element.

1. Introduction

Direct analysis or second-order inelastic analysis is a useful tool for practical design and investigation of structural behavior under ultimate limit states and has been extensively researched over the past few decades for conventional and performance-based design, studies for progressive collapse, structural fire engineering and so on. In order to accurately model the holistic structural behavior and stability, the vital effects inherent to a real building are needed to be considered and these include initial imperfections, residual stress and geometric and material nonlinearities.

The beam-column element analysis method is generally regarded to be considerably more efficient and effective in the design of practical framed structures and extensive research has been conducted since the 1970s. Much effort has been made by numerous researchers, including Meek and Tan [1], Chan and Kitipornchai [2], Chan [3], Bridge et al. [4], Chan and Zhou [5], Chen and Chan [6], Izzudin and Smith [7], Spacone et al. [8], Izzuddin [9], Liew et al. [10], Neuenhofer and Filippou [11], Pi et al. [12,13] and so on. Numerical techniques have been extensively used for solution of related nonlinear engineering problems.

The analytical model using one element per member for advanced, direct analysis and second-order analysis and design of steel frames has been investigated by various researchers [14,15]. This technique not only brings convenience and efficiency in design, but also reduces difficulties of modeling member initial imperfection, which is essential and crucial for safe design using either the second-order elastic or the inelastic analysis.

Initial imperfections are present in all structural members and frames, which significantly affect the stability and strength. The importance of proper modeling of initial imperfections has been reported by Chan et al. [16,17]. The analysis model based on one element per member is highly preferred and considered as a fundamental requirement for practical, efficient and accurate modeling of frames allowing for effects of initial imperfection. The directions of the initial member imperfections are generally set in the same shape as the global buckling mode or the load deflection mode in order to generate the more adverse effects. Consequently, the technique to model one member by several elements not only increases the computational time, but also brings much inconvenience to updating of the model in each case of load combination. To this end, the element formulation for modeling the initial member imperfection by one element is desired and proposed in the past decade.

Conventionally, the plastic hinge approach assumes plasticity occurred at a certain short length near the two ends of the element such that the locations of two lumped plastic hinges are assumed at either end and the associated rotational stiffness is deteriorated for simulation of the gradual cross-section yielding. Many researchers have successfully adopted this method for inelastic analysis of steel frames, such as Al-Mashary and Chen [18], White [19], Kim and Chen [20], Chan and Chui [21], Chiorean and Barsan [22], Gong [23] and so on. However, in some circumstances, the plastic hinge could be formed at a zone other than the ends of the member. Therefore, two or more elements are required to model each member which causes much inconvenience in modeling the member initial imperfections and increases computer time.

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With this background, a curved beam-column element, which allows plastic hinges to form in an arbitrary location along the member as well as two end zones, is proposed to allow the direct analysis to be more applicable in practical design. Also, the mathematical formulation and modeling procedure for the refined plastic hinge will be given in this paper. In order to incorporate directly the proposed element into existing non-linear structural analysis software, the stiffness condensation approach and the generalized external nodal forces are derived and reported herein. Finally, several benchmark examples are employed to validate the proposed theory.

2. Assumptions

The following basic assumptions are adopted in the current study as:

1) the Euler–Bernoulli hypothesis is valid and the member is mainly under axial loads and the second-order effect due to axial loads is considered;
2) strains are small but the deformation can be large;
3) plane sections before deformation remain planar after deformation which implies that a linear strain distribution exists across the section depth;
4) material nonlinearity is considered by plastic hinge spring while the element remains elastic;
5) the applied loads are nodal with distributed load lumped to element nodes and conservative, which are assumed to be independent of the load path and proportional increase and;
6) warping deformation, shear deformation and twisting effects are not considered.

The above assumptions are in line with conventional assumption for design of common civil engineering structures and requirements of national design codes. Further, lateral–torsional buckling is considered as a local effect and can be carried out by checking against standard and empirical formulae in codes. This design consideration is carried out independently and therefore will not lead to limitations in the use of the proposed element. Further, prevention of lateral–torsional buckling is generally taken as a pre-requisite condition for plastic design so it is not generally a controlling criterion in plastic design of practical structures.

3. Element formulations for planar analysis

3.1. Background

In this paper, a new curved beam-column element with arbitrarily located hinge (ALH element) is proposed and it is graphically shown in Fig. 1 and developed from the three-hinge beam element proposed by Chen and Chan [6], with the improvements summarized as (1) the location of internal hinge can be arbitrary and not necessarily at mid-span as assumed in Chen and Chan [6]; (2) member initial imperfections are taken into account; and (3) the plastic hinge model is refined for the modeling of gradual yielding.

In element formulation, two sets of coordinate systems are introduced (i.e., a fixed global co-ordinate and a local convective system) as illustrated in Fig. 2. In the local convective system, the member deflections are separated from the nodal translations so that the element stiffness deviation can be simplified and concise.

Due to the presence of the internal nodes, there are three more additional degrees of freedom (DOFs) in the proposed element in comparison with the conventional beam-column element in a two-dimensional space. To incorporate the element efficiently into an existing computer program, the internal DOFs will be condensed before global stiffness assembly and decoupled in calculating the resisting forces. The member forces and DOFs in these coordinate systems are shown in Fig. 2.

3.2. Formulation of displacement function

The force versus displacement relations of the arbitrarily located hinge (ALH) element are illustrated in Fig. 1. The initial imperfection is assumed as:

$$v_0 = v_{mo}\frac{\left(L^2-4x^2\right)}{L^2} \quad \text{and} \quad -L/2 \le x \le L/2 \tag{1}$$

where v_0 is the shape function of initial member imperfection; v_{mo} is the amplitude of initial imperfection at mid-span; L is the length of the member; x is the distance along the element (see Fig. 1).

It is noted that, the finite element interpolation is a piecewise function, which can be expressed with eight coefficients of a_i and b_i with i = 0 to 3 given by,

$$v = \begin{cases} v_1 = a_0 + a_1x + a_2x^2 + a_3x^3 \\ v_2 = b_0 + b_1x + b_2x^2 + b_3x^3 \end{cases} \quad \text{and} \quad \begin{matrix} -L/2 \le x \le \xi L \\ \xi L \le x \le L/2 \end{matrix} \tag{2}$$

where v is the lateral displacement function due to applied loads; ξ is the location of the internal plastic hinge; a_i and b_i are the coefficients for the polynomial function which will be determined by the boundary conditions.

In order to solve the eight coefficients in the shape function, eight boundary conditions are introduced as follows,

$$\begin{cases} v = v_1 = 0 \\ \dot{v} = \dot{v}_1 = \theta_{11} \end{cases} \quad \text{at} \quad x = -L/2 \tag{3}$$

$$\begin{cases} v = v_1 = v_2 = \delta \\ \dot{v} = \dot{v}_1 = \theta_{12} \\ \dot{v} = \dot{v}_2 = \theta_{21} \end{cases} \quad \text{at} \quad x = \xi L \tag{4}$$

$$\begin{cases} v = v_2 = 0 \\ \dot{v} = \dot{v}_2 = \theta_{22} \end{cases} \quad \text{at} \quad x = L/2 \tag{5}$$

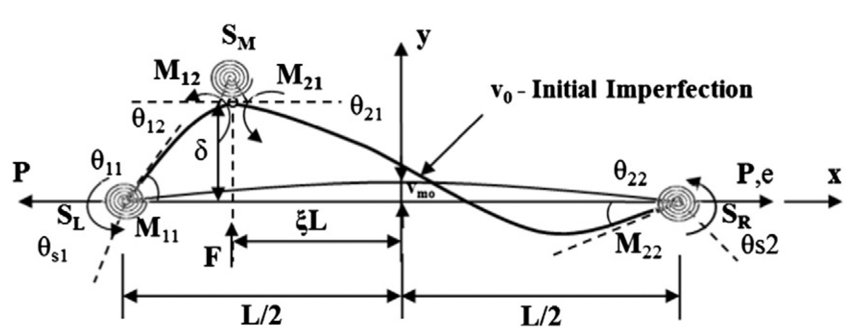

Fig. 1. The force versus displacement relations of the ALH element.