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# Exact solution of bending problem for continuous sandwich panels with profiled facings

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ABSTRACT

The paper presents a closed-form solution, in the distributional form, of the bending problem for continuous sandwich panels in lightweight cladding that has been derived from the general solution of the system of differential equations for the problem of sandwich beams with thick facings. The analysis covers three-layer members with facings made from profiled metal sheets and a core having good insulating properties, for any number and spacing of elastic intermediate supports. Panels are subjected to bending under uniform transverse load and a load caused by temperature difference. The solution for the multi-span panels inflexibly (rigidly) pin-supported at their ends is derived from the general solution for members arbitrarily supported at their ends. A calculation algorithm for this type of panels is provided. Numerical examples are included to illustrate the influence of the intermediate supports on the effort of both the entire sandwich panel and its profiled facing. Finally, practical concluding remarks are given.



## 1. Introduction

The failure of sandwich panels with low-profiled thin metal facings and a core having good insulating properties (polystyrene foam, foamed polyurethane, mineral wool), used in the lightweight cladding of building structures, occurs due to the instability of the compression facing, in the shape of local buckling (wrinkling). Fig. 1 shows examples (from the authors' own research [1]) of such a failure of sandwich panels with steel sheet skins and a polyurethane core. This failure mechanism has recently been studied in, e.g., [2–7]. The bending resistance of sandwich panels, due to local buckling effects can be considerably increased by the use of facings from deep-profiled (trapezoidal) sheets [8], Fig. 2.

The places where the compression facing of sandwich panels wrinkles, correspond usually to the panel cross-sections with extreme bending moments. The values of the latter depend on both the external load and the adopted static scheme and they are determined by the global static analysis of continuous sandwich panels. This can be done, with some limitations, using the appropriate tables included in the standard [9] and in specialist literature [10–12]. The limitations are discussed in [13] where closed solutions on bending of continuous thin faced sandwich panels (considered as appropriate beams on arbitrarily spaced elastic supports, segmentally uniformly transversely loaded or loaded with a temperature difference) were derived. The solutions [13] can be used to develop simple algorithms for the static analysis of sandwich panels by means of, e.g., Mathcad or even scientific calculators.

The current paper presents closed-form, distributional solutions of the problems relating to the bending of continuous sandwich panels with thick facings, substituted by adequate beams on arbitrarily spaced elastic supports, uniformly transversely loaded or loaded by temperature difference. The solutions have similar advantages as the ones presented in [13]. Practically, the solutions are easier to apply in static calculations than the classical methods of solving statically indeterminate elements (forces, displacements, three or four moments, etc.), described in, e.g., [11,12,14,15]. It should be noted that the access to computer software for the calculation of sandwich members is rather limited [15,16]. It is also known that besides global static analyses, the analysis of the local effort of the facings in the zones of supports and joints is of major importance [11,17]. The effect of local bending can be reduced if the proper support widths recommended by the manufacturers of the sandwich panels are used.

## 2. Differential equations of bending for sandwich panels with profiled facings

The model for the considered sandwich panels under bending is based on 1.00 m wide thick faced sandwich beams whose cross-section is shown in Fig. 3. The static scheme of the beams with pin-supported ends is shown in Fig. 4. The external load is represented by uniformly distributed transverse load $q$ [kN/m] and constant temperature difference $\Delta T$ [°C].

In the case of a three-layer beam having width $b$ (Fig. 3), with a core having thickness $h$ and characteristics: $A_c, E_c, G_c$ (the cross-sectional area, the Young's modulus and the shear modulus of the core, respectively)

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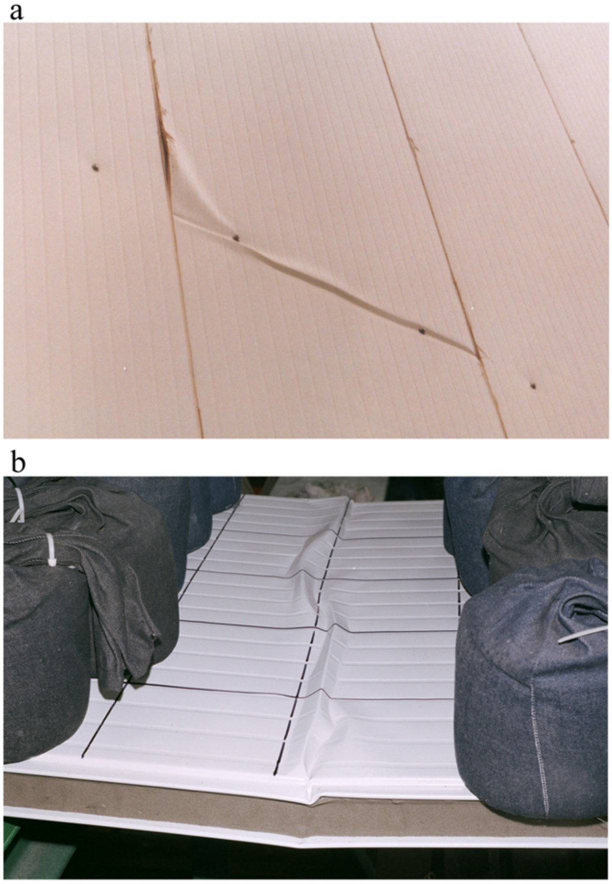


Fig. 1. Sandwich wall panels with low-profiled facings made of steel sheet after exceedance of bending resistance: a) in the course of operation and b) during tests.

and with facings having thicknesses $h_{ex}$, and $h_{in}$ (external and internal faces, respectively) and characteristics: $A_{ex}$, $A_{in}$, $E$, $I_{ex}$, $I_{in}$ (the cross-sectional area, the Young's modulus and the second moments of area of the facings relative to their axes, respectively), made from deep-profiled sheet with thicknesses $t_{ex}$, and $t_{in}$ ($h_{ex,\,in} > 10t_{ex,\,in}$), the differential equations for bending line $w(x)$ and core shear angle $\gamma(x)$, respectively, can be written, Hop [18] and Davies [11] as:

$$-\frac{B_{ex}+B_{in}}{B_c}w^{(6)}+\frac{B}{B_s}w^{(4)}=\frac{1}{B_s}p-\frac{1}{B_c}p^{(2)}-\vartheta^{(2)}, \quad (1)$$

$$-\frac{B_{ex}+B_{in}}{B_c}\gamma^{(4)}+\frac{B}{B_s}\gamma^{(2)}=-\frac{1}{B_c}p^{(1)}-\frac{B_{ex}+B_{in}}{B_c}\vartheta^{(3)}, \quad (2)$$

Fig. 2. Roof covering from sandwich panels with deep-profiled upper facing and low-profiled bottom facing.

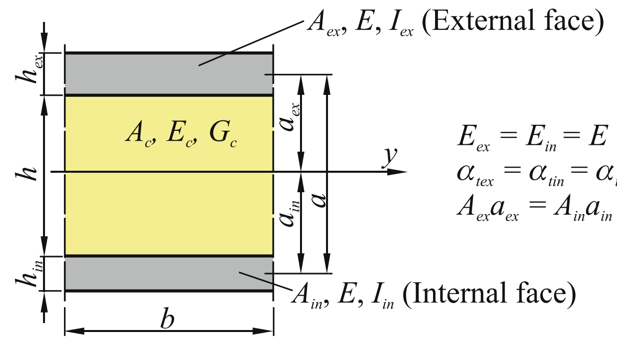


Fig. 3. Cross-section and designation of three-layer panel (beam) with thick faces.

where: $B_{ex}$, $B_{in}$—the bending stiffness of the facings ($B_{ex,\,in} = EI_{ex,\,in}$, respectively), $B_c$—the shear stiffness of the core ($B_c = G_c ba^2/h$), $a$—distance between centers of gravity of external and internal faces, $B$—the bending stiffness of the sandwich panel ($B = B_{ex} + B_{in} + B_s = B_f + B_s$), $B_s$—the stiffness resulting from the laminar structure

$$B_s = \frac{EA_{ex}\cdot EA_{in}}{EA_{ex}+EA_{in}}a^2 = \frac{EA_{ex}\cdot A_{in}}{A_{ex}+A_{in}}a^2,$$

$p$—the transverse load, $\vartheta = \alpha_t \cdot \Delta T/a$ ($\Delta T = T_{in} - T_{ex}$—the temperature difference between the internal and external faces, $\alpha_t$—the coefficient of thermal expansion of the facings), and $\xi^{(k)} = d^{(k)}\xi(x)/dx^k$, ($\xi$=$w$, $p$, $\gamma$, $\delta$, $H$, $\vartheta$). The other denotations are explained in Fig. 3.

Expressions (1) and (2) are ordinary differential equations with constant coefficients of the sixth and fourth order, respectively. At this stage they do not constitute a system of equations, and can be solved independently. The solutions are used to determine the internal forces in sandwich panels (beams) from the following relationships [11,18]:

- bending moments

$$M(x) = B_s\left(\gamma^{(1)}-\vartheta\right)-Bw^{(2)}, \quad (3)$$

- shear forces

$$V(x) = B_c\gamma-(B_{ex}+B_{in})w^{(3)}. \quad (4)$$

Differential Eqs. (1) and (2) can be written in brief:

$$-\frac{B_f}{B_c}w^{(6)}+\frac{B}{B_s}w^{(4)}=\frac{1}{B_s}p-\frac{1}{B_c}p^{(2)}-\vartheta^{(2)}, \quad (5)$$

$$-\frac{B_f}{B_c}\gamma^{(4)}+\frac{B}{B_s}\gamma^{(2)}=-\frac{1}{B_c}p^{(1)}-\frac{B_f}{B_c}\vartheta^{(3)}. \quad (6)$$

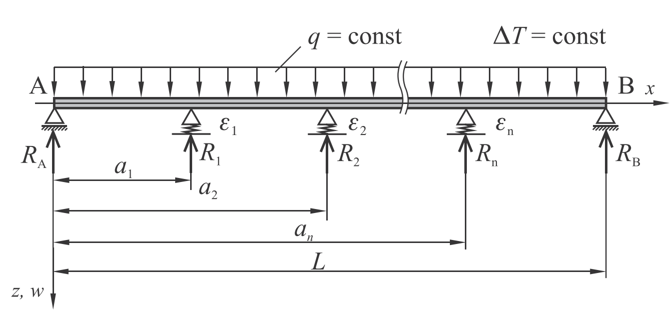


Fig. 4. General scheme and designation of continuous sandwich panel (beam) uniformly, transversely loaded or loaded by constant temperature difference.