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Extension of EC3-1-1 interaction formulae for the stability verification of tapered beam-columns

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ABSTRACT

EC3 provides several methodologies for the stability verification of members and frames. Regarding tapered beam-columns, in EC3-1-1, the safety verification may be performed by the General Method. However, application of this method has been shown not to be reliable. On the other hand, the interaction formulae in EC3-1-1 were specifically calibrated for stability verification of prismatic members.

Recently, Ayrton–Perry based proposals for the stability verification of web-tapered columns and beams, in line with the Eurocode principles for the stability verification of prismatic members, have shown to lead to a substantial increase of accuracy and to provide mechanical consistency relatively to application of the General Method. Such methodologies may be further applied to the existing interaction formulae.

It is the purpose of this paper to propose a verification procedure for the stability verification of web-tapered beam-columns under in-plane loading by adaptation of the interaction formulae in EC3-1-1, validated through extensive FEM numerical simulations covering several combinations of bending moment about strong axis, M_y, and axial force, N, and levels of taper.

1. Introduction

Tapered members are used in structures mainly due to their structural efficiency, providing at the same time aesthetical appearance. Examples of the application of tapered steel members in various structures are given in Figs. 1.1 and 1.2.

Tapered members are commonly applied in steel frames, namely industrial halls, warehouses, exhibition centers, etc. Adequate verification procedures are then required for these types of structures. Some structural configurations are illustrated in Fig. 1.3.

In the scope of member design, maximum taper ratios (defined as the ratio between the maximum and the minimum height of the tapered member – $\gamma_h = h_{max}/h_{min}$) of $\gamma_h = 4$ may be assumed to cover a large proportion of existing structures. This issue will most likely be more pronounced for cross sections with slender webs (class 4) for which a higher variation may be observed. However, in this study, since only global member failure modes were the scope of analysis, the authors assumed this limit for the proposed rules. Interaction between local and global failure modes will be studied in a next step of the research. Figs. 1.1 to 1.3 illustrate taper ratios within that range of $\gamma_h < 4$, even for the shorter members.

EC3 provides several methodologies for the stability verification of members and frames. Regarding non-uniform members in general, with tapered cross-section, irregular distribution of restraints, nonlinear axis, castellated, etc., several difficulties are noted regarding the stability verification. Moreover, there are yet no guidelines to overcome these issues. As a result, to ensure safety, over-conservative verification is likely to be performed by the designer, not accounting for the advantages and structural efficiency that non-uniform members may provide.

In EC3-1-1 [1], the safety verification of a tapered beam-column may be performed by the General Method; by a second order analysis considering all relevant imperfections followed by a cross section check; or by a numerical analysis taking account of all relevant nonlinear geometrical and material effects.

The last two options involve adequate numerical modeling to incorporate the second order effects, which is, for the time being, not the preferred alternative as the definition of a wide combination of relevant imperfections is not always simple to consider. The alternative of using the General Method is also not very reliable for the following reasons:

- The General Method requires that the in-plane resistance of the member accounting for second order in-plane effects and imperfections is considered as an absolute upper bound of the member resistance ($\alpha_{ult,k}$). The consideration of in-plane (local and global) imperfections for the determination of the in-plane load multiplier $\alpha_{ult,k}$ of the General Method may result in a need to perform complex numerical analyses, as there are yet no analytical stability verification procedures for non-uniform members;

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Nomenclature

Lowercases

a_0, a, b, c, d	Class indexes for buckling curves according to EC3-1-1
a_γ	Auxiliary term to the taper ratio for application of LTB proposed methodology
b	Cross section width
e_0	Maximum amplitude of a member imperfection
f_y	Yield stress
h	Cross section height
h_{max}	Maximum cross section height
h_{min}	Minimum cross section height
$h_{xcII,lim}$	Cross section height at $x^{II}_{c,lim}$
k_{yy}, k_{zy}, k_{yz}, k_{zz}	Interaction factors dependent of the phenomena of instability and plasticity involved
n	Number of cases
t_f	Flange thickness
t_w	Web thickness
$x^{II}_{c,lim}$	Second order failure cross section for a high slenderness level
$x^i_{c,N}$, $x^i_{c,M}$, $x^i_{c,MN}$	Denomination of the failure cross section (to differentiate from the type of loading it refers to): N — do to axial force only; M — due to bending moment only; MN — due to the combined action of bending moment and axial force
x^I_c	First order failure cross section
x^{II}_c	Second order failure cross section
x_{min}	Location corresponding to the smallest cross section
x-x	Axis along the member
y-y	Cross section axis parallel to the flanges
z-z	Cross section axis perpendicular to the flanges

Uppercases

A	Cross section area
A_{min}	Cross section area of the smallest cross section in of a tapered member
C_m	Equivalent moment factor according to clause 6.3.3
CoV	Coefficient of variation
E	Modulus of elasticity
FEM	Finite Element Method
GM	General Method
GMNIA	Geometrical and Material Non-linear Analysis with Imperfections
L	Member length
LBA	Linear Buckling Analysis
LTB	Lateral Torsional-Buckling
M	Bending moment
$M_{b,Rd}$	Design buckling resistance moment
M_{Ed}	Design bending moment
$M_{f,Rd}$	Cross section resistance to bending considering the area of the flanges only
MNA	Materially Non-linear Analysis
$M_{pl,y,Rd}$	Design value of the plastic resistance to bending moments about *y-y* axis
M_y	Bending moments, *y-y* axis
$M_{y,Ed}$	Design bending moment, *y-y* axis
N	Normal force
$N_{cr,z}$	Elastic critical force for out-of-plane buckling
N_{Ed}	Design normal force
N_{pl}	Plastic resistance to normal force at a given cross section
$N_{pl,Rd}$	Design plastic resistance to normal forces of the gross cross section
UDL	Uniformly distributed loading

Lowercase Greek letters

α	Angle of taper
α, α_{EC3}	Imperfection factor according to EC3-1-1
$\alpha_b^{(Method)}$	Load multiplier which leads to the resistance for a given method
α_{cr}	Load multiplier which leads to the elastic critical resistance
$\alpha_{cr,op}$	Minimum amplifier for the in-plane design loads to reach the elastic critical resistance with regard to lateral or lateral–torsional buckling
α^M_{pl}	Load amplifier defined with respect to the plastic cross section bending Moment
α^N_{pl}	Load amplifier defined with respect to the plastic cross section axial force
$\alpha_{ult,k}$	Minimum load amplifier of the design loads to reach the characteristic resistance of the most critical cross section
γ_{M1}	Partial safety factor for resistance of members to instability assessed by member checks
δ_0	General displacement of the imperfect shape
δ_{cr}	General displacement of the critical mode
ε	Utilization ratio at a given cross section
ε^I_M	Utilization ratio regarding first order bending moment *M*
ε^{II}_M	Utilization ratio regarding the second order bending moment
ε_N	Utilization ratio regarding the axial force *N*
η	Generalized imperfection
$\overline{\lambda}_{op}$	Global non-dimensional slenderness of a structural component for out-of-plane buckling according to the general method of clause 6.3.4
$\overline{\lambda}$	Non-dimensional slenderness
$\overline{\lambda}(x)$	Non-dimensional slenderness at a given position
$\overline{\lambda}_y$	Non-dimensional slenderness for flexural buckling, *y-y* axis
$\overline{\lambda}_z$	Non-dimensional slenderness for flexural buckling, *z-z* axis
$\overline{\lambda}_{LT}$	Non-dimensional slenderness for lateral–torsional buckling
$\overline{\lambda}_{LT,0}$	Plateau length of the lateral torsional buckling curves for rolled sections
$\overline{\lambda}_0$	Plateau relative slenderness
φ	Over-strength factor
ϕ	Ratio between α^M_{pl} and α^N_{pl}
φ_y, φ_z, φ_{LT}	Over-strength factor for in-plane buckling, out-of-plane buckling, lateral–torsional buckling
χ	Reduction factor
χ_{LT}	Reduction factor to lateral–torsional buckling
χ_{num}	Reduction factor (numerical)
χ_{op}	Reduction factor for the non-dimensional slenderness $\overline{\lambda}_{op}$
χ_y	Reduction factor due to flexural buckling, *y-y* axis
χ_z	Reduction factor due to flexural buckling, *z-z* axis
χ_z	Reduction factor to weak axis flexural buckling
ψ	Ratio between the maximum and minimum bending moment, for a linear bending moment distribution
ψ_{lim}	Auxiliary term for application of LTB proposed methodology

- It has been proven that, for the case of columns (even prismatic), this assumption for the determination of $\alpha_{ult,k}$ leads to conservative estimates of the ultimate resistance (up to 20%) [9]. In addition, it leads to also overly conservative resistance if the in-plane effects are