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Elastic instability and free vibration analyses of tapered thin-walled beams by the power series method

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ABSTRACT

In this paper, the flexural-torsional buckling and free vibration of tapered thin-walled beam-columns with arbitrary cross-section shape are extensively investigated. The governing equilibrium equations and motion equations are obtained from the stationary condition of the potential energy. The strain energy is derived in presence of initial stresses. In the work of the applied forces, effects of load eccentricities from shear and centroid centerlines are taken into account. Free vibration is considered in presence of harmonic excitations. In presence of arbitrary boundary conditions and variable cross-section properties, a semi-analytical approach based on power series method is adopted in solution. According to this method, displacements and geometric constants are approximated by polynomial functions up to a certain order, where accurate results are reached. The flexural-torsional buckling loads or natural frequencies are determined by solving an eigenvalue problem. In order to measure the accuracy and to check the validity of the present method, several examples including flexural-torsional behavior and free vibration analysis of non-prismatic thin-walled members with web and flange tapering and various boundary conditions are considered. The obtained results are compared to the finite element simulations and other available solutions.

1. Introduction

The use of thin-walled beams with I, Z, C and Angle cross sections has been increasing in many steel construction and mechanical components due to their ability to utilize structural material more efficiently and optimize the distribution of weight. Flexural-torsional stability and vibration analyses are important topics in the design of these structures and research of accurate models has interested many researchers over the world from the last mid century until now. Moreover, prediction of the buckling loads and vibration modes of thin-walled beams with arbitrary cross-section is complex due to flexural-torsional coupling and warping presence. The task seems to be more complicated in presence of tapered thin-walled elements where the cross-section properties are not constant.

Since the early works of Timoshenko and Vlasov [1,2], the extensive investigations in static and dynamic behavior of thin-walled beam members have noticeably increased their understanding. In [3,4], the buckling loads of prismatic beams are evaluated either by closed-form solutions of the fourth-order differential equations governing the twisting and the bending equilibrium or by using of complementary energy principle. Different numerical procedures, mainly based on the finite element method, have been developed for the stability and vibration analyses of prismatic thin-walled beams with singly symmetric or arbitrary cross-section shapes [5–18]. Based on the classical variational principle and the theory for thin-walled shells, Zhang [5] provided a model for the flexural-torsional buckling of thin-walled members under simply supported boundary conditions using shell elements. The previous model has been recently extended to cantilever beams [6]. Erkmen [7,8] developed a finite element model for the buckling analysis of thin-walled members based on principle of stationary complementary energy and Koiter's polar decomposition theory. Wu and Mohareb [9,10] derived theoretical and finite element models for buckling analysis of shear deformable beam-column elements with doubly symmetric I cross sections. Effects of load positions relative to the shear and centroid centerlines are incorporated. The exact dynamic stiffness matrix of straight members are derived by Leung [11], using power series expansion. General distributed axial forces are considered and the interaction diagrams due to vibration frequency, axial force and torque are studied. Kim [12,13] proposed an improved numerical method, based Vlasov's assumption and Hellinger–Reissner principle and derive static and dynamic stiffness matrices for the lateral-torsional buckling and vibration analyses of thin-walled beam with non-symmetric section subjected to linear variable axial force. Numerical solutions are made possible by adoption of the power series method. Jun [14] derived the dynamic transfer matrix for a straight uniform and axially loaded thin-walled Bernoulli–Euler beam element and used it in calculation of exact natural frequencies and mode shapes of the non-symmetrical thin-walled beams. Borbon [15] developed a coupled torsional-flexural finite element for vibration analysis thin-

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walled beam with non-symmetric cross-section including the effects of rotatory inertia, shear deformation and eccentric axial load. Previous studies were developed for linear stability context. Based on non-linear model, Mohri [16] developed a finite element approach for open section beams in large torsion context. In [17], the 3-factor formula used in the lateral buckling stability is discussed and extended to cross-sections with large flanges where pre-buckling deflections are important. Based on the previous model, buckling and free vibration interaction were studied in Mohri [18].

Mentioned studies were focused on the stability and vibration analyses of prismatic thin-walled beams. However, in the last decades, thin-walled beams with variable cross-sections are extensively adopted in different steel constructions. Therefore, their vibration and stability analyses gained more attention by many authors. The investigations of elastic flexural-torsional buckling and vibration modes of tapered thin-walled with the use of efficient numerical techniques such as finite element method in the validation process have attracted many researchers. Among the first investigations on this topic, the most important ones are the studies of Yang [19] who formulated a finite element model for the beam that takes into account the effect of non-uniform torsion. The effect of geometric non-linearity is considered and the updated Lagrangian approach is adopted in solution. Pasquino [20] used a variational approach to derive the Euler–Lagrange equations to model non-prismatic thin-walled beams with arbitrary cross-section and evaluate their buckling loads. More recently, Kim [21] investigated the linear stability and free vibration behavior of doubly symmetric I tapered thin-walled beams by a finite element approach. A theoretical model for large torsion context and equivalent beam finite element formulation in finite torsion amplitudes were developed by Ronagh [22,23]. Effects of non-linear terms like flexural-torsional coupling in presence of cubic terms of torsion angle were considered and applied to the case of doubly symmetric tapered cross-sections. Chen [24] applied Hamilton's principle to the potential energy and obtained the motion equations governing of non-prismatic thin-walled beams with arbitrary cross-sections. An equilibrium differential equation considering both the effects of shear deformation and constant axial force of tapered beams was established by Li [25]. Chebyshev polynomial approach was adopted in solution of the second-order differential equation with variable coefficients. Recently, Yau [26] determined the linear elastic and geometric stiffness matrices of a torsionally loaded I-beam element with tapered cross-section by considering the non-uniform torsions and the second-order effects of warping moments. A general variational formulation to analyze the lateral-torsional buckling behavior of singly symmetric cross-section tapered beams was presented by Andrade [27,28]. In the study, Rayleigh–Ritz method is applied in the case of simply supported and cantilever beams and comparisons to shell elements are made in the validation process.

Most of previous works devoted to non-prismatic beams have been limited to either doubly symmetric or singly symmetric cross-sections in both buckling and vibration analyses. In Asgarian [34], the lateral-torsional behavior of tapered beams with singly symmetric I cross-sections was studied. The equilibrium equations were discretized by the power series expansions method. The previous work is extended to include eccentrically axial forces and dynamic loads for beams with non-symmetric cross-sections and arbitrary boundary conditions. Load positions on the cross section contour are taken into consideration. For this purpose, the equations of motion are derived from the energy principle of the thin-walled beam subjected to bending and axial loads. In presence of non-prismatic thin-walled beam-columns with arbitrary cross-sections, the flexural-torsional displacements and section properties are highly dependent on the axial coordinate. In order to make the solution of the problem possible, the power series expansions are used to solve the fourth-order differential equations of motion of the non-prismatic thin-walled beam. In this regard, it is assumed that the functions which describe the member's variable parameters such as: flexural rigidity, cross-section area and density can be expanded into power series form. Based on the aforementioned method, the expressions of the bending displacements and the torsion angle are also presented into power series form. The critical buckling loads and natural frequencies can be obtained by imposing the natural boundary conditions corresponding to thin-walled members under combined effect of bending and torsion and solving the eigenvalue problem.

In order to demonstrate the accuracy and efficiency of this method, some numerical examples are presented. The obtained results are compared to shell finite element results using Ansys software and to other available numerical and analytical investigations. The proposed study can be applied for stability and free vibration analyses of various forms of non-prismatic thin-walled members. The proposed method can be used for the buckling and free vibration analyses of uniform beams as well as non-uniform members.

2. Derivation of equilibrium and motion equations for buckling and free vibration

2.1. Kinematics

A tapered thin-walled column with arbitrary cross-section is considered in the study (Fig. 1a). The length $\boldsymbol{L}$ of the thin-walled beam is larger compared to the cross-section dimensions. A direct rectangular co-ordinate system is chosen, with x as the initial longitudinal axis and y and z as the first and second main bending axes. The origin of these axes is located at the centroid O. The shear center with co-ordinates (y_c, z_c) in xyz is denoted in C (Fig. 1b). For flexural torsional buckling stability, the member is subjected to arbitrary distributed forces p_x, p_y and p_z in structural domain in x, y and z directions respectively. Displacement components of point M on the section contour can be expressed in terms of those of origin O. Displacement components are denoted by U, V and W. The torsion angle is denoted by θ. The following relationships are commonly used:

$$U(x,y,z,t) = u_0(x,t) - y\frac{\partial v_0(x,t)}{\partial x} - z\frac{\partial w_0(x,t)}{\partial x} - \phi(y,z)\frac{\partial \theta(x,t)}{\partial x} \quad (1)$$

$$V(x,y,z,t) = v_0(x,t) - z\theta(x,t) \quad (2)$$

$$W(x,y,z,t) = w_0(x,t) + y\theta(x,t) \quad (3)$$

In which U represents the axial displacement. The displacement components V and W represent lateral and vertical displacements (in y and z directions). In Eq. (1), the term $\phi(y,z)$ is the warping function, which can be defined based on Saint Venant's torsion theory. The two components of vertical and lateral displacement at the centroid can be replaced by the displacements at the shear center C as follows:

$$v_0(x,t) = v(x,t) + z_c(x)\theta(x,t) \quad (4)$$

$$w_0(x,t) = w(x,t) - y_c(x)\theta(x,t). \quad (5)$$

The Green's strain tensor components which incorporate the large displacements and including linear and non-linear strain part are given by:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial U_k}{\partial x_i}\frac{\partial U_k}{\partial x_j}\right) = \varepsilon_{ij}^l + \varepsilon_{ij}^* \quad (6a)$$

where $i, j, k = x, y, z$

ε_{ij}^l denotes the linear part and ε_{ij}^* denotes the quadratic non-linear part. Substituting Eqs. (4) and (5) into Eqs. (1)-(3), using Green strain tensor Eq. (6a) and taking into account for tapering, the linear strain components for the thin-walled beam can be obtained as:

$$\varepsilon_{xx}^l = u_0' - y\left(v'' + z_c\theta'' + z_c''\theta + 2z_c'\theta'\right) - z\left(w'' - y_c\theta'' - y_c''\theta - 2y_c'\theta'\right) - \phi\theta'' \quad (6b)$$