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Flexural buckling of laced column with fir-shaped lattice

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ABSTRACT

In contrast to the classical Engesser method of solving the buckling problem for laced columns in terms of an "equivalent" solid bar, the buckling problem of a column with a fir-shaped lattice is formulated as a stability problem of a statically indeterminate system of elastic bars. Solving this problem by conventional methods consists of the determination of a smallest eigenvalue for the linear algebraic system of a high order which depends upon the number of the column joints. The present approach requires analyzing only the fourth-order system for columns with any degree of static indeterminacy. The stability analysis is reduced to numerical solution of a two-point boundary value problem for a system of recurrence relations between deformation parameters of column cross-sections passing through the column joints. The critical force and the modified slenderness ratio for column with any number of panels and the fixed inclination of lattice diagonals are represented as a function of the lattice rigidity parameter. The obtained values of Euler's critical force are essentially higher than those obtained with Engesser's model. The distinctive feature which is similar to the Boobnov phenomenon occurs for the column with a fir-shaped lattice: the column loses stability so that joint cross-sections are not displaced and the chord panels are buckled as a simply-supported bar. This type of buckling is possible when the lattice rigidity exceeds a specific limit. The plots of the modified slenderness ratio as a function of the lattice rigidity can be applied in designing steel-laced columns with a fir-shaped lattice.

1. Introduction

According to design codes the nominal compressive strength of steel columns is determined by using column strength curves developed on the concept of the limit state of flexural buckling. The equations describing the strength curves depend on the ratio between the yield stress of the column material and the elastic buckling stress [1]. So the problem of elastic buckling is a significant point in determining the strength of metal columns.

The laced columns with a fir-shaped lattice are the highly redundant structures. In design practice, determination of the Euler critical load for such columns in the direction parallel to the lattices [2–5] is based on the Engesser's hypothesis which reduces the problem to calculation of Euler's critical load for the "equivalent" solid pin-ended bar while the effect of shear deformation on the bar deflections is taken into account [6,7]. An extension of the Engesser's method proposed in [8,9] is based on the assumption that the transverse displacement of the column chords is the same and that the column lattices behave as a shear panel continuously connected to the chords. To achieve a more satisfactory value of critical load, an application of the general theory of stability of a system of elastic bars is necessary [7]. The critical load for a laced column is found by equating to zero the determinant of the system of homogeneous linear equations representing the equilibrium conditions of the column joints in a slightly deflected state. The determinant can be of high order since it depends upon the number of the column joints. However, a buckling problem for a laced column with any number of panels can be formulated as a two-point boundary value problem for the eighth-order system of recurrence relations between deformation parameters of column cross-sections passing through the column joints. These relations result from formulas of the initial-value method for the centrally compressed solid elastic bar. A determination of the critical force and buckling mode shape is reduced to finding the smallest eigenvalue for the fourth-order system of linear algebraic equations. Rigidity properties of the fir-shaped lattice are described by rigidities of diagonal and brace. The Euler's critical force of the column can be represented as a function of the lattice rigidity parameter and the number of the column panels. The present method has been previously used for solving the stability problem of laced columns with serpentine and crosswise lattices [10–12] and built-up columns with batten plates [13].

2. Initial-value method

The deflection w of an elastic straight bar subjected to an axial compression by the force P in the critical state is described by the well-known differential equation

$$EIw^{IV} + Pw^{II} = 0 \qquad (1)$$

where E is Young's modulus and I is the second moment of inertia of the cross-sectional area [7]. The roman superscripts denote a differentiation with respect to the axial longitudinal coordinate z. The bending moment

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Notation

A	cross-sectional area of the column chord
A_b	cross-sectional area of the horizontal lattice brace
A_d	cross-sectional area of the lattice diagonal
a	length of the chord panel (Fig. 1)
c_b	rigidity of the horizontal lattice brace
c_d	rigidity of the lattice diagonal in the direction normal to the column chords
E	Young's modulus
I	second moment of inertia of the cross-section
i	number of the column chord, $i = 1, 2$
k	number of the joint cross section of the column, $k = 0, 1, 2,\ldots, n$
M	bending moment
$M_i(k)$	bending moment at the joint cross section k of the ith chord
N_a	Euler's critical force for a simply-supported bar geometrically similar with the chord panel
n	number of panels in the column chord
P_{cr}	Euler's critical value of the axial compressed force P applied to the column chord
Q	transverse force
$Q_i(k)$	transverse force at the joint cross section k of the ith chord
$Q_i(k,\uparrow)$	transverse force at the cross section with coordinate $z = z_k + 0$ of the ith chord
$Q_i(k,\downarrow)$	transverse force at the cross section with coordinate $z = z_k - 0$ of the ith chord
$R_{ib}(k)$	lateral force applied to the joint cross section k of the ith chord from the horizontal lattice braces
$R_{id}(k)$	lateral force applied to the joint cross section k of the ith chord from the lattice diagonals
$\mathbf{V}$	vector of deformation parameters in the system of recurrence equations
$\mathbf{V}_j(k)$	particular solution of recurrence equations at the joint cross section $k, j = 1, 2, 3, 4$
$\widetilde{\mathbf{V}}_j(k)$	orthonormalized vector of deformation parameters at the joint cross section $k, j = 1, 2, 3, 4$
w	transverse displacement of the compressed elastic bar
$y_i(k)$	transverse displacement of the joint cross section k of the ith chord
z	coordinate along the axis $O_i Z$ (Fig. 1)
z_k	z-coordinate for the joint cross sections k of both chords
α_d	non-dimensional rigidity parameter of the lattice diagonal
α_b	non-dimensional rigidity parameter of the horizontal lattice brace
α_B	magnitude of the parameter α_d which gives rise to the Boobnov phenomenon
$\Gamma_i = \frac{M_i a}{EI_i}$	non-dimensional bending moment at the chord cross section
$\Lambda_i = \frac{Q_i a^2}{EI_i}$	non-dimensional transverse force at the chord cross section
φ	inclination angle of the lattice diagonal to the column cross-section (Fig. 1)
λ_a	slenderness ratio of the chord panel
λ_m	modified slenderness ratio of the laced column
$\nu = \sqrt{P/(EI)}$	coefficient of the action of compressive force P on the bar deflection
υ	inclination of the chord cross section in the deformed state
$\xi_i = \frac{y_i}{a}$	non-dimensional transverse displacement of the chord cross section
$\Omega_i = \sqrt{\frac{P_i}{EI_i}}a$	non-dimensional parameter of the compressive force P_i

M and the transverse force Q directed normally to the bar axis in the initial unstressed state are expressed as follows:

$$M = -EIw^{II} \tag{2}$$

$$Q = -EIw^{III} - Pw^{I}. \tag{3}$$

A solution of Eq. (1) can be expressed in terms of initial values of deflection w and its derivatives by using the initial-value method which makes possible to represent w, w^I, M and Q in the following form [14]:

$$\begin{aligned} w &= w_0 + \left(w^I\right)_0 \frac{\sin\nu z}{\nu} - \frac{M_0}{EI}\frac{1-\cos\nu z}{\nu^2} - \frac{Q_0}{EI}\frac{\nu z - \sin\nu z}{\nu^3} \\ w^I &= \left(w^I\right)_0 \cos\nu z - \frac{M_0}{EI}\frac{\sin\nu z}{\nu} - \frac{Q_0}{EI}\frac{1-\cos\nu z}{\nu^2} \\ M &= EI\left(w^I\right)_0 \nu \sin\nu z + M_0 \cos\nu z + Q_0 \frac{\sin\nu z}{\nu} \\ Q &= Q_0 \end{aligned} \tag{4}$$

where $\nu = \sqrt{P/(EI)}$ and w_0, $(w^I)_0$, M_0 and Q_0 are initial values of deflection, its derivative, bending moment and transverse force, respectively.

3. Reactions of diagonals and braces at the lattice joints

Consider a laced column, which consists of two longitudinal solid chords 1 and 2 connected by two mutually parallel fir-shaped lattices in one (Fig. 1). The column cross-section is symmetric about the axis parallel to the lattice planes. In actual practice, the fir-shaped lattice is one of the most frequently employed lattice types. The lattice consists of the ascending diagonals that start at the joints located on the left chord and of horizontal braces. The lattice joints make up hinges in the lattice plane. We shall distinguish identical quantities corresponding to the different chords by means of subscripts $i = 1, 2$. Each chord is compressed by an axial force P_i. Introduce an ordinal numeration $k = 0, 1, 2,\ldots, n$ for joint cross-sections of each chord, where n is the number of column panels. A length of panel can vary along the column. However, we have denoted the panel length as a.

Associate a local Cartesian coordinate system with each chord. It is formed by the principal axis of inertia O_iY of the cross-section and by the longitudinal axis O_iZ. We use the symbol z for the coordinate along the axis O_iZ. The joints located on the different chords differ in construction. We assume that the joint cross-section of the left chord is located between the horizontal brace and the diagonal that ascends from the joint, and the joint cross-section of the right chord is located between the diagonal that arrives at the joint and the horizontal brace (Fig. 2). However, the values of z-coordinate for the joint cross-sections k of both chords are taken to be the same

$$z_1(k) = z_2(k) = z_k. \tag{5}$$

We denote the transverse displacement, its derivative, and the bending moment for the joint cross-section k of the ith chord as follows:

$$y_i(k) = y_i(z_k), y_i^{\,I}(k) = y_i^{\,I}(z_k), M_i(k) = M_i(z_k). \tag{6}$$