Journal of Constructional Steel Research 95 (2014) 290–299

Contents lists available at ScienceDirect

Journal of Constructional Steel Research

Out-of-plane elastic buckling behavior of hinged planar truss arch with lateral bracings

Yan-lin Guo, Si-Yuan Zhao *, Chao Dou

Department of Civil Engineering, Tsinghua University, Beijing 100084, PR China

ARTICLE INFO

Article history:
Received 29 November 2012
Accepted 21 December 2013
Available online 29 January 2014

Keywords:
Planar truss arch
Hinged
Elastic
Buckling
Out-of-plane
Lateral bracings

ABSTRACT

This paper is concerned with the out-of-plane elastic buckling behavior of planar truss arch with continuous or discrete lateral bracings. Firstly, planar truss arches with continuous lateral bracings are studied, and a closed form solution of out-of-plane elastic buckling load is obtained theoretically. As for planar truss arches laterally restrained with discrete bracings, two key parameters, namely lateral bracing threshold stiffness and out-of-plane elastic buckling load of an arch segment between two adjacent bracings, are investigated by the finite element method, respectively. Besides, it is found from finite element results that, if the planar truss arch is manufactured by using intersecting joints between web members and chord tubes and is laterally restrained on top chord tubes, the out-of-plane buckling of bottom chord tubes will fully be prohibited. However, the gusset-plate connections between chord tubes and web members cannot prevent the planar truss arch from the out-of-plane buckling due to their weak stiffness, even though the arch is laterally fully restrained on top chord tubes.

1. Introduction

Arches in building structures could be classified as solid-web section arches, planar truss arches and spatial truss arches, as shown in Fig. 1. The latter two have a higher load-carrying capacity than the former one because they transfer the sectional moment into axial forces of chords and web members, and are widely used in long-span structures.

As we know, arches loaded in-plane and unrestrained laterally may suddenly deflect and twist out of the plane of loading, usually under a load much lower than the in-plane buckling load. Therefore, continuous lateral bracings or discrete lateral bracings may be arranged appropriately along the arch axis. Continuous lateral bracings are uniformly distributed along the arch axis and have a close spacing between adjacent bracings, such as the roof sheeting as shown in Fig. 2a and b, while spacing between two adjacent bracings may be quite large for discrete lateral bracings as shown in Fig. 2c.

Classical solutions for elastic buckling loads of circular arches with a solid-web section under uniform bending or under uniform compression have been studied extensively [1–10]. However, researches on the out-of-plane buckling of arches with lateral bracings have rarely been reported. Pi and Bradford [11] studied the effect of continuous lateral bracings on the out-of-plane buckling load of solid-web section arches by using an energy method. As to discrete lateral bracings, Bradford and Pi [12] investigated the out-of-plane elastic buckling behavior of solid-web section arches with one lateral bracing located on the crown of arch. For planar truss arches with continuous lateral bracings or discrete lateral bracings, no studies have been reported so far.

This paper is concerned with the out-of-plane elastic buckling of circular planar truss arches subjected to uniformly distributed radial loads. Continuous lateral bracings or discrete lateral bracings are uniformly distributed along the arch axis, providing the lateral-translational restraints to the out-of-plane displacement at the bracings point. Arch is assumed to be simply supported at both ends, namely the translations in three directions and twist rotation along arch axis which are fully restrained (Fig. 3a). However, when a circular arch is subjected to uniformly distributed radial loads, to ensure that it is in uniform compression, the translations at two ends along radial direction of arch should be released in the analysis (Fig. 3b).

Firstly, planar truss arches in uniform compression with continuous lateral bracings located on the top chord tubes or the bottom chord tubes are investigated, and the closed form solution of out-of-plane buckling load is obtained by using linear elastic buckling analysis. As for planar truss arches with discrete lateral bracings, two key design parameters, namely the bracing threshold stiffness value and out-of-plane buckling load of an arch segment between two adjacent bracing points, are discussed by the finite element method (FEM) to verify the out-of-plane stability of the arch. In addition, for a planar truss arch fully restrained on the top chord tubes or the bottom chord tubes, it may suddenly buckle with the bottom chord tubes or the top chord tubes deforming laterally (Fig. 4), and this will be studied in this paper as well.

Planar truss arches to be investigated in this paper are shown in Fig. 5, where S is the developed length of the arch axis; h is the height of the planar truss (defined as the distance from the top chord tube axis to the bottom chord tube axis); L is the span of the arch; f is the

* Corresponding author. Tel.: +86 01062788124.
E-mail address: zhaosy005@163.com (S.-Y. Zhao).

0143-974X/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jcsr.2013.12.013

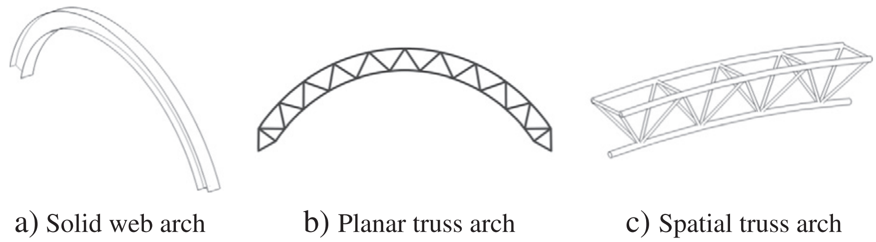

Fig. 1. Types of arches.

rise of the arch; and the dimensionless parameter f/L denotes the rise-to-span ratio and varies from 0.15 to 0.4 in this paper. Finite element models are developed with BEAM188 for all the components of truss arches in commercial FE software package ANSYS [13], which is based on Timoshenko beam theory. The lowest eigenvalues obtained by ANSYS respond to the out-of-plane elastic buckling load of planar truss arches. Accordingly, the pre-buckling deformation and geometric imperfections have not be involved in the analysis. Lateral bracings of planner truss arches are characterized by their elastic stiffness of spring.

2. Planar truss arches with continuous lateral bracings

Continuous lateral bracings such as the roof sheeting system on the top chords or on the bottom chords could significantly raise the out-of-plane buckling load of arches (Fig. 2a–b). If the arches with continuous lateral bracings have their out-of-plane buckling loads greater than the in-plane buckling loads, as expressed by Eq. (1), the buckling verification of arches is only limited to in-plane instability.

$$q_{\mathrm{cr,out}} \geq q_{\mathrm{cr,in}} \tag{1}$$

In the above Eq. (1), $q_{\mathrm{cr,out}}$ is the out-of-plane buckling load intensity of arches, depending on the continuous lateral bracing arrangements and their stiffness k_{t}, and $q_{\mathrm{cr,in}}$ is the in-plane buckling load intensity of arches. If the truss arch is subjected to uniformly distributed radial loads q on top chords or bottom chords, as shown in Fig. 6, the in-plane buckling load $q_{\mathrm{cr,in}}$ is deduced by Guo & Guo [14]:

$$\begin{cases} q_{\mathrm{cr,in}} = \dfrac{\overline{q}_{\mathrm{cr,in}}}{1+\overline{q}_{\mathrm{cr,in}}\overline{R}/K_v} \\ \overline{q}_{\mathrm{cr,in}} = \dfrac{EI_x}{\overline{R}^3}\left(\dfrac{4\pi^2}{\Theta^2}-1\right) \end{cases}. \tag{2}$$

In Eq. (2), E is the Young's modulus of elasticity; if q is located on the top chords, $\overline{R}=R+h/2$, else if q is located on the bottom chords, $\overline{R}=R-h/2$, where R is the radius of arch axis; Θ is the arch included angle (Fig. 6); I_{x} is the major axis second moments of cross section; and K_{v} is the in-plane shear stiffness of section, and is given by:

$$\begin{cases} I_x = \dfrac{1}{2}A_1 h^2 \\ K_v = EA_2\sin^2\theta\cos\theta \end{cases} \tag{3}$$

where A_1 is the cross-sectional area of chord tubes; A_2 is the cross-sectional area of web members; and θ is the angle between chord tubes and web members (Fig. 6).

Out-of-plane spatial equilibrium equations of planar truss arch are derived by authors [15] as:

$$\begin{cases} \dfrac{d^2M_y}{d\varphi^2}+NR\left(\dfrac{d^2u}{Rd\varphi^2}+\psi\right)-q_xR^2+\dfrac{dM_z}{d\varphi}=0 \\ \dfrac{dM_z}{d\varphi}-M_x\left(\dfrac{d^2u}{Rd\varphi^2}+\psi\right)-M_y=0 \end{cases} \tag{4}$$

in which, N, M_{x}, M_{y} and M_{z} represent the axial compression force, the in-plane bending moment, the out-of-plane bending moment and the torsional moment on the section of the planar truss arch, respectively; u is the out-of-plane displacement of centroid of the cross-section, ψ is the twist rotation of the cross-section, q_{x} is the component of external distributed load q along axis o–x, and φ is the coordinate of central angle, as shown in Fig. 6.

For planar truss arches subjected to uniformly distributed radial loads and restrained laterally by continuous bracings on the top chord tubes (Fig. 7), the related parameters of Eq. (4) are given by:

$$\begin{cases} N = q\left(R+\dfrac{h}{2}\right) \\ q_x = q\psi-k_t\left(u+\dfrac{h\psi}{2}\right) \\ M_x = 0 \end{cases}. \tag{5}$$

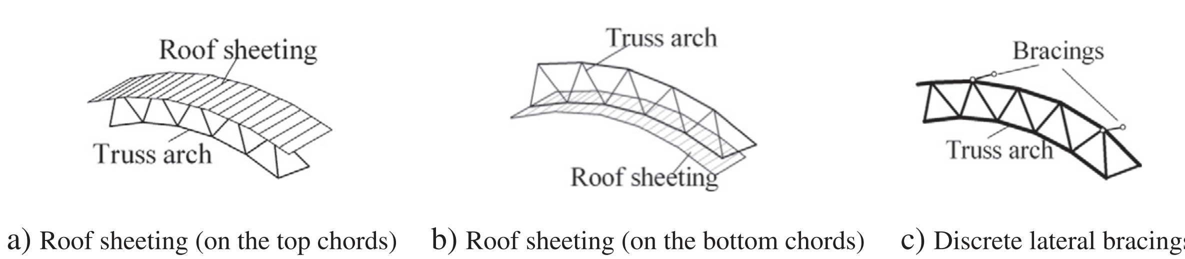

Fig. 2. Lateral bracings of arches.