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Imperfection sensitivity of column instability revisited

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ABSTRACT

The buckling of columns is the classic problem in structural stability. It has been studied by many researchers over a large number of years, and it is well known that the severity of the buckling response can be greatly amplified by initial geometric imperfections in the column shape. The current paper presents and discusses the effects of imperfection shape, orientation and magnitude on the buckling behavior of columns. Analyses are conducted for elastic columns with overall initial imperfections in the form of out-of-straightness and sway displacements, as well as local imperfections that, for instance, model constructional and material defects. Traditionally, the initial imperfections are modeled with the first buckling mode with a size selected according to fabrication tolerances. This approach will not necessarily provide a lower limit to the column pre-buckling stiffness and strength. These assertions are supported by numerical results for imperfection-sensitive columns. The influence of end restraint on column strength is also studied since columns in actual frameworks are connected to other structural members such that their ends are restrained.

1. Introduction

The pioneering investigation by Euler on the elastic stability of a mathematically straight, prismatic, pin-ended, concentrically loaded, slender column founded the development of the “classical” theory of elastic buckling, in which the governing equations of the problem are linear and homogeneous in the lateral displacement w and its first three derivatives, leading typically to a eigen-boundary-value problem [1]. The critical buckling loads are the eigenvalues and the corresponding buckling mode shapes are the eigenvectors of the problem. Practical columns, however, deviate from this ideal model due to the inevitable presence of geometrical, material, structural and load related imperfections that act to reduce the column load-carrying capacity. For modeling purposes, geometrical, material and structural imperfections are usually considered by means of an equivalent initial geometric imperfection, in the form of an initial curvature or out-of-straightness (lateral deflection of the column relative to the undeformed state) and an initial sway imperfection (relative lateral displacement between the column ends) [2–4].

The buckling load of a compressed ideal column is also affected by the boundary conditions. For all possible boundary conditions, the critical load can be always related to the basic pin-ended column element through the concept of the *effective length*, L_{eff} that was first introduced by Jasinsky in 1893 [5]. The effective length is defined as the length of a pin-ended column that has the same critical load as a column with other prescribed end-conditions. This concept allows codes of practice to be simplified considerably — see, for instance, the American specification for structural steel buildings [6] and the European code of practice for the design of steel structures, EN 1993 [7], with rules only necessary for pin-ended columns. In fact, much of the research on elastic buckling of columns is based on the behavior of this simple column. In particular, and among the many analyses of imperfection sensitive columns, the majority is concerned with the buckling of a simple uniform column with equivalent initial geometric imperfections and eccentric axial loads. Most studies consider an initial centerline deflection in the form of a half-sine curve, $\delta_0 \sin(\pi x/L_{eff})$, where δ_0 is the initial out-of-straightness at the middle of the pin-ended equivalent column. This is a simplified form of geometric imperfections that uses a single buckling mode representation of the imperfection in which the amplitude δ_0 is allowed to vary according to the fabrication tolerances stipulated in the applicable engineering standards [8], or in some cases being treated as a random variable [9,10].

The equivalent geometric imperfections change the column response. An imperfect column exhibits a limit point with bending being introduced from the onset of loading. Since the bending effect is likely to be secondary compared to the effect of the axial force, it is current practice to assume that the characteristic column strength is reached when the material yield stress f_y is first attained at any point in the column; in the

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context of the Ayrton–Perry formulation [11], the maximum stress σ_{max} is given by:

$$\sigma_{max} = \frac{N_{Ed}}{A} + \frac{M_{max}}{W_{el}} \leq f_y \qquad (1)$$

where N_{Ed} is the applied axial compressive load, A is the area of cross section, $W_{el} = Ai^2/c$ is the section modulus corresponding to the fiber with maximum elastic stress with i being the section radius of gyration and c being the distance from neutral axis to extreme fiber, and M_{max} is the maximum bending moment in the column. The theory that effectively combines the Ayrton–Perry approach to failure of columns with a global imperfection parameter, as proposed by Robertson [12] for the definition of the buckling resistance of columns, is essentially elastic in nature [11] and leads to the well-known *column curves* based on the *effective length* concept that are adopted in modern design codes. The EN 1993 document, in particular, adopts a single equation for the column design curves developed by Rondal and Maquoi [13]; these describe the practical compressive strength of a column in terms of the slenderness ratio $\bar{\lambda} = \sqrt{N_{pl}/N_{cr}}$, where N_{cr} is the elastic critical load and N_{pl} is the squash load.

The important problem of determining consistent equivalent geometric imperfections includes the choice of their shape and size and its relation to the column end restraints. The equivalent initial imperfections are usually assumed to be proportional to the classical buckling mode (or as a linear combination of the relevant critical modes), but in fact these are functions involving uncertainties that may have a random nature. In principle, the choice of the imperfection shape that leads to the lowest column strength subjected to amplitude constraints should involve a Fourier-type approach, which usually gives a good interpretation of the actual column imperfections [9].

With this range of issues in mind, the research presented currently has the following specific objectives:

1. To dissociate the form of initial geometric imperfections from the traditional approach of assuming the imperfection shape affine to the lowest bifurcation mode while bounded by a given imperfection amplitude.
2. To include the effect of a localized deflection pattern within the problem domain (i.e. a localized geometric imperfection to a section of the column). Such localized effects may arise in steel columns, for example, due to welding of a reinforcing steel plate over a short column length, which would modify the residual stress distribution, for example. This effect is then combined with the general distributed imperfection pattern along the column length to compare the strength ratios to the critical load.
3. To examine the influence of end restraint on the column strength and behavior. More specifically, two aspects of restrained column behavior are considered: the magnitudes of the rotational and sway end stiffnesses, plus the form and bow amplitude of the initial geometric imperfections.
4. To compare the effects of the various imperfections and end support conditions by using the maximum elastic strength as the relative measure and then derive design sensitivities of the critical load factor for the various studies. A parametric study is conducted through computational analysis and results are then established.

As far as the authors are aware, little attention has been paid to this class of problems, and the physical results are in themselves of interest. In particular, it is shown that for a given imperfection amplitude, the shape of such an imperfection is an important factor in the column strength ratios to the buckling load. These effects are examined with exact treatments through extensive computational modeling.

The buckling formulation used, as well as the underlying computational implementation, can be easily extended to other types of structures. For example: thin-plated structures, columns on elastic foundations, sandwich struts and prestressed stayed columns; the sensitivity to the geometry of the imperfection of the latter three components has been investigated to some extent already [14–16].

2. Elastic buckling of uniform columns

The elastic buckling of a perfect column is a classic textbook problem, see [2,3,17]. Columns that are very slender do not have a significant sensitivity to initial geometric imperfections; only when yielding comes into the picture does the sensitivity show itself. This is significant for moderately slender columns. In that range of slenderness, an imperfect column has a load-carrying capacity that is usually significantly less than that obtained from calculating the buckling load of a column with a perfect geometry.

2.1. Basic equations

Consider an elastically supported column with initial deformations associated with out-of-straightness of the column, $w_{\delta 0}(x)$, and initial sway imperfection, Δ_0 (see Fig. 1). The following assumptions are made:

1. The analysis is purely elastic with the stress–strain relationship being completely linear and E is defined as the Young's modulus.
2. The column is of uniform cross-section with I being the second moment of area.
3. The column is stress-free in its initial configuration, before the application of the axial load.

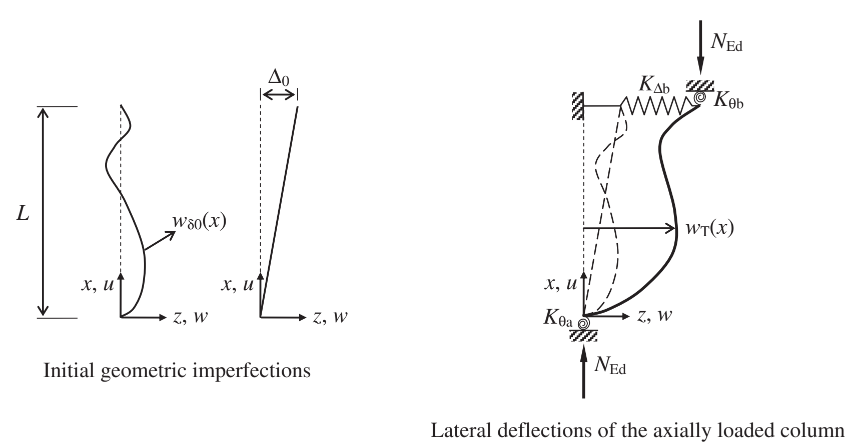

Fig. 1. Elastically supported column with equivalent geometric imperfections.