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# Resistance of steel I-sections under axial force and biaxial bending

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ABSTRACT

The plastic criteria for the verification of steel cross-sections resistance are usually based on some basic hypotheses such as the development of plastic hinges, which depend on the interaction between the internal forces and the cross-section shape; therefore, specific equations are required for each type of cross-section.

This paper presents new alternative interaction criteria for the analysis of steel I-sections subjected to an axial force and biaxial bending moments, at the elastic or the plastic limit states (as long as buckling phenomena are not involved).

The plastic interaction criteria are presented, in a first step, for some particular combinations of the internal forces, such as axial loading with bending about a main axis, and biaxial bending without axial loading. In these cases, they are given by exact equations (within the frame of the hypotheses adopted in this study). All these plastic interaction criteria are compared with the corresponding plastic criteria adopted in the Eurocode 3 (EC3).

Afterwards, a simplified global criterion is proposed for the simultaneous combination of an axial force and bending moments about both the main axes of inertia. This new simplified plastic criterion and the corresponding plastic criterion adopted in the EC3 are compared with the exact solution, obtained by a mixed numerical and analytical integration procedure. This comparison shows that this simplified criterion usually leads to results closer to the exact solutions. Some suggestions are then presented to improve the results given by the EC3.



## 1. Introduction

The analysis of the behaviour and limit carrying capacity of a cross-section under biaxial bending is usually a complex problem, which has been studied by many researchers for a long time. A large number of publications may be found, covering the study of structural cross-sections made of different materials (such as reinforced concrete sections [14,29], composite steel-concrete sections [25,26], steel sections [31], or aluminium sections [13], for instance). A review of different methods used for the evaluation of the cross-sections plastic resistance may be found in [27]; most of them essentially consider only axial stresses due to axial loading and biaxial bending for the determination of the plastic section capacity. Warping normal stresses due to bimoments, as well as shear stresses from bending, uniform torsion and warping are either disregarded or considered only approximately in some of those approaches [27].

In the case of steel sections, a considerable amount of research has been done concerning the study of different types of cross-sections, such as H and I shapes [20], solid and hollow rectangular sections [18,31], or angle sections [33,34]. Some extensive reviews of these research works may be found in several publications, such as [15] or [20] for instance.

The elastic–plastic methods are currently adopted in modern standard codes of design to estimate the ultimate resistance of some steel structures, since they allow the beneficial effects of yielding in the redistribution of stresses to be taken into account. The analysis of the limit carrying capacity of a cross-section under biaxial bending is simpler than the analysis of its behaviour along the elastic–plastic range, [6,9], and hence the earliest papers were restricted to that problem [35].

The research works carried out with this purpose have been based on analytical studies [16,19], experimental investigations [13,28,32], and numerical models [12,21,22]. A large number of these studies took in account other aspects than the elastic or plastic carrying capacity of the cross-sections, such as the possible occurrence of local or overall buckling phenomena of the structural elements in biaxial bending [28,33,34].

Although the results of some numerical models evidence a very good agreement with test results, their practical use for design purposes is limited, since most of them are not currently available and the labour required by the numerical calculations is quite important [22]. Therefore, their applications usually remain within the limits of research studies, and the designers often rely on simple interaction equations, between the cross-section internal forces, which may be found in bridge and building specifications such as [1–5,17], for instance.

The interaction criteria between the cross-section internal forces at its plastic limit state depend on the cross-section shape. Consequently, specific analytical expressions are required for each type of cross-sections. However, these analytical expressions are not currently available for some cross-section shapes, or they are defined by

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means of simplified equations, which do not take in account all the possible scenarios of loading, depending on the combinations of internal forces and relevant geometrical parameters.

On the other hand, the existing accurate methods are frequently complex, and difficult to apply in practice. This is often the case when biaxial bending of a cross-section is involved.

The design interaction formulae used to check the safety of members and cross-sections subjected to biaxial bending and axial force are usually the result of previous research studies, which are in the origin of those formulae or were dedicated to their discussion and validation. One of these interaction criteria, indicated in Eq. (1), was proposed by Bresler [14] and it has been adopted as the basis of the most common design criteria stated in the structural codes, for the verification of different types of cross-sections (solid and hollow rectangular sections, or H and I-shapes for instance) made of different materials, such as steel, aluminium, reinforced concrete, composite concrete and steel, etc.:

$$\left(\frac{M_{n,y}}{M_{o,y}}\right)^{\alpha_1} + \left(\frac{M_{n,z}}{M_{o,z}}\right)^{\alpha_2} = 1.0 \quad (1)$$

where $M_{n,y}$ and $M_{n,z}$ are the bending moment components, about the cross-section main axes of inertia, associated to an axial load $N$, and $M_{o,y}$ and $M_{o,z}$ represent the cross-section resistance capacities in simple bending under the axial load $N$, when $M_{n,z}=0$ or $M_{n,y}=0$ respectively. Many solutions have been suggested for the evaluation of the $\alpha_1$ and $\alpha_2$ coefficients or for alterations to Eq. (1) [25], in order to adjust it to the ultimate resistance capacity of different cross-section shapes and materials [17].

Rubin [30] has proposed new interaction criteria between the bending moment, the shear force and the axial force for simple symmetrical box and I-sections, when bent about their strong axis, and for double-symmetric I-sections bent about their weak axis. These equations are in the basis of the specifications from the Eurocode 3 [3] and from the German Steel Code DIN 18800 [4,5], for specific section types such as I-sections, circular tubes, rectangular hollow sections and solid rectangles and plates [27].

Yet, even if these equations give a good estimation of the cross-section resistance for a large number of practical situations, some research works have pointed out its limitations and have presented alternative solutions, namely under the form of design tables [19].

This work presents new interaction criteria for the analysis of I-shaped cross-sections subjected to a combination of an axial force and biaxial bending moments, at the elastic or plastic limit states (as long as buckling phenomena are not involved). Written in a non-dimensional form, these criteria are independent from the cross-section dimensions and steel strength, and from the Unit System used in the analysis [6,7]. The main advantage of these interaction criteria lies on a better approach of the real cross-section resisting internal forces, when compared with the results given by other simplified criteria, such as those adopted on the EC3 design code. Next, some suggestions are presented to improve the results given by the EC3 criteria.

## 2. Basic principles of the analytical criteria

### 2.1. Assumptions

Fig. 1 presents a general configuration of an I-shaped cross-section under biaxial bending. The $v$ axis is assumed to be the bending axis. Its direction is defined by the cross-section linear segment where the stresses due to biaxial bending are equal to zero.

The cross-section $b$ dimension represents its width, parallel to the $y$ axis, and the $h$ dimension corresponds to its height, parallel to the $z$ axis (Fig. 1); $h_w$ is the web height; $t_w$ and $t_f$ are the web and the flanges thicknesses, respectively (Fig. 1).

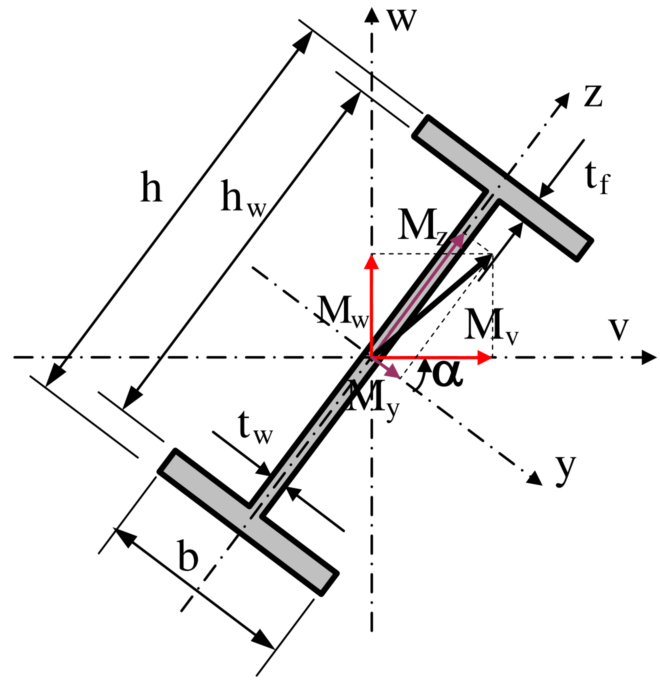


**Fig. 1.** Symbols and reference axes.

The values of $M_y$ and $M_z$ are supposed to be always positive; therefore, the inclination angle $\alpha$ of the bending axis $v$ regarding the $y$ axis is within the limits $0 \leq \alpha \leq \pi/2$.

In the case of uniaxial bending about the strong axis, we have $M_w=0$, $M_z=0$, $M_y=M_v$ and $\alpha=0$; if the bending axis is the weak axis, we have $M_w=0$, $M_y=0$, $M_z=M_v$ and $\alpha=\pi/2$ (Fig. 1).

### 2.2. Strain distribution

The distribution of the cross-section longitudinal strains (associated to its normal stresses) is based on the classical Bernoulli hypothesis that, after deformation, the cross-sections remain plane and normal to the structural element longitudinal axis. Therefore, the strain field may be defined by the following expression:

$$\varepsilon(y,z) = \varepsilon_N - \chi_y z + \chi_z y \quad (2)$$

where $\varepsilon_N$, $\chi_y$ e $\chi_z$ represent the cross-section global deformations (axial deformation $\varepsilon_N$, and bending curvatures $\chi_y$ and $\chi_z$, about the cross-section main axes); $y$ and $z$ are the coordinates of a cross-section point regarding the same cross-section main axes.

The cross-section neutral axis is parallel to the bending axis $v$, and it is defined by:

$$\varepsilon(y,z) = 0 \Rightarrow z = \frac{1}{\chi_y}\ (\varepsilon_N + \chi_z y) \quad (3)$$

Therefore:

$$\mathrm{tg}\,\alpha = \frac{\partial z}{\partial y} = \frac{\chi_z}{\chi_y} \quad (4)$$

The change between the coordinate systems associated to the $(v,w)$ axes or to the $(y,z)$ axes may be carried out by means of the following equations:

$$\begin{cases} v = y\cos\alpha + z\,\mathrm{sen}\,\alpha \\ w = -y\,\mathrm{sen}\,\alpha + z\cos\alpha \end{cases} \quad (5)$$