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Bending analysis of partially restrained channel-section purlins subjected to up-lift loadings

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ABSTRACT

Cold-formed steel section beams are widely used as the secondary structural members in buildings to support roof and side cladding or sheeting. These members are thus commonly treated as the restrained beams either fully or partially in its lateral and rotational directions. In this paper an analytical model is presented to describe the bending and twisting behaviour of partially restrained channel-section purlins when subjected to uplift loading. Formulae used to calculate the bending stresses of the roof purlins are derived by using the classical bending theory of thin-walled beams. Detailed comparisons are made between the present model and the simplified model proposed in Eurocodes (EN1993-1-3). To validate the accuracy of the present model, both available experimental data and finite element analysis results are used, from which the bending stress distributions along the lip, flange and web lines are compared with those obtained from the present and EN1993-1-3 models.

1. Introduction

Thin-walled, cold-formed steel sections are widely used in buildings as sheeting, decking, purlins, rails, mezzanine floor beams, lattice beams, wall studs, storage racking and shelving. Among these products, purlins and rails are the most common members, widely used in buildings as the secondary members supporting the corrugated roof or wall sheeting and transmit the force to the main structural frame. Roof purlins and cladding rails have been considered to be the most popular products and account for a substantial proportion of cold-formed steel usage in buildings.

In the UK, most common sections are the zed, channel and sigma shapes, which may be plain or have stiffened lips. The lips are small additional elements at free edges in a cross section, and so added to provide the structural efficiency under compressive loads [1]. Roof purlins and sheeting rails are usually restrained against lateral movement by their supported roof or wall cladding. Such restraints reduce the potential of lateral buckling of the whole section, but do not necessarily eradicate the problem [2]. For example, roof purlins are generally restrained against lateral displacement by the cladding, but under wind uplift, which induces compression in the unrestrained flange, lateral-torsional buckling is still a common cause of failure [3]. This occurs due to the flexibility of the restraining cladding and to the distortional flexibility of the section itself, which permits lateral movement to occur in the compression flange even if the other flange is restrained.

Several researchers have investigated the behaviour of the roof purlins with partial restraints provided by their supported cladding or sheeting. For example, Lucas et al. investigated the interaction between the sheeting and purlins using finite element analysis methods [4,5]. Ye et al. presented several examples to demonstrate the influence of sheeting on the bending [6], local and distortional buckling behaviour [7] of roof purlins. Vieira et al. provided simplified models to predict the longitudinal stresses when the channel-section purlin is subjected to uplift loading [8]. The lateral-torsional buckling of purlins subjected to downwards and/or upwards loadings has also been discussed by several researchers [9–13]. Analytical models have been developed to predict the critical loads of lateral-torsional buckling and the influence of sheeting on the lateral-torsional buckling behaviour of roof purlins [12–14]. Experimental tests have also been performed on both bridged and unbridged zed- and channel-section purlins under uplift loads [15,16]. Calculation models for predicting the rotational restraint stiffness of the sheeting have been proposed recently [17,18]. Design specifications for the purlin-sheeting system have been provided in Eurocodes [3].

In this paper an analytical model is presented to describe the bending and twisting behaviour of the partially restrained channel-section purlins when subjected to uplift loading. The classical bending theory of thin-walled beams is used to calculate the bending stresses of the roof purlins. In order to validate the model, both available experimental data and finite element analysis results are used, from which the bending stress distributions along the lip, flange and

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web lines are compared with those obtained from the present and EN1993-1-3 models.

2. Analytical model

Consider a channel section that is partially restrained by the sheeting on its upper flange. When the member is subjected to a uniformly distributed uplift load acting on the middle line of the upper flange, the restraint of the sheeting to the member can be simplified as a lateral restraint and a rotational restraint. For most types of sheeting the lateral restraint is sufficiently large and therefore the lateral displacement at the upper flange-web junction may be assumed to be fully restrained. The rotational restraint, however, is dependent on the dimensions of sheeting and purlin, number, type and positions of the screws used in the fixing. If the stiffness of the rotational restraint provided by the sheeting is known, then the purlin-sheeting system may be idealized as a purlin with lateral displacement fully restrained and rotation partially restrained at its upper flange-web junction as shown in Fig. 1.

Let the origin of the coordinate system (*x,y,z*) be the centroid of the channel cross-section, with *x*-axis being along the longitudinal direction of the beam, and *y*- and *z*-axes taken in the plane of the cross-section, as shown in Fig. 1. According to the bending and torsion theory of beams [1,19], the equilibrium equations, expressed in terms of displacements, are given as follows,

$$EI_z\frac{d^4v}{dx^4}=q_y \tag{1}$$

$$EI_y\frac{d^4w}{dx^4}=q_z \tag{2}$$

$$EI_w\frac{d^4\phi}{dx^4}-GI_T\frac{d^2\phi}{dx^2}+k_\phi\phi=z_kq_y-y_qq_z \tag{3}$$

where v and w are the *y*- and *z*-components of displacement of the cross-section defined at the shear centre, ϕ is the angle of twisting of the section, E is the modulus of elasticity, G is the shear modulus, I_y and I_z are the second moments of the cross-sectional area about *y*- and *z*-axes, I_w is the warping constant, I_T is the torsion constant, k_ϕ is the per-unit length stiffness constant of the rotational spring, q_y and q_z are the densities of the uniformly distributed loads in *y*- and *z*-directions, z_k is the vertical distance from the shear centre to the force line q_y, and y_q is the horizontal distance from the shear centre to the force line q_z.

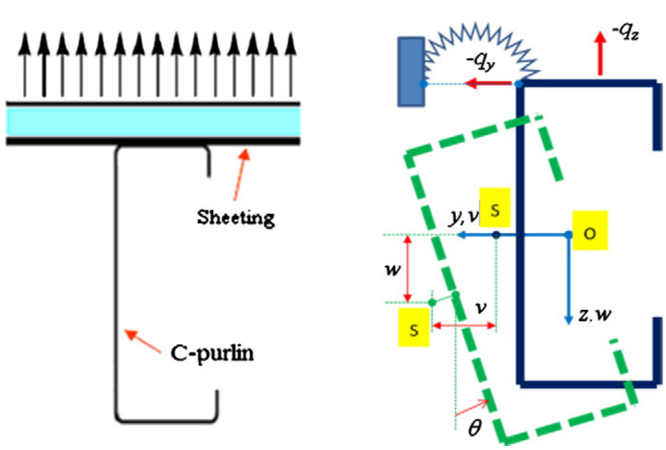

Fig. 1. Analytical model used for a channel-section purlin-sheeting system.

Using Eq. (1) to eliminate q_y and Eq. (2) to eliminate w in Eq. (3), it yields,

$$EI_w\frac{d^4\phi}{dx^4}-GI_T\frac{d^2\phi}{dx^2}+k_\phi\phi-z_kEI_z\frac{d^4v}{dx^4}=-y_qq_z \tag{4}$$

Note that, the lateral displacement restraint applied at the upper flange-web junction requires,

$$z_k\phi+v=0 \tag{5}$$

Using Eq. (5) to eliminate the angle of twisting, ϕ, in Eq. (4), it yields,

$$\left(I_z+\frac{I_w}{z_k^2}\right)\frac{d^4v}{dx^4}-\frac{GI_T}{Ez_k^2}\frac{d^2v}{dx^2}+\frac{k_\varphi}{Ez_k^2}v=\frac{y_qq_z}{Ez_k} \tag{6}$$

Let

$$a_0=\frac{k_\phi}{Ez_k^2} \tag{7}$$

$$a_1=\frac{GI_T}{Ez_k^2} \tag{8}$$

$$a_2=I_z+\frac{I_w}{z_k^2} \tag{9}$$

With the use of Eqs. (7)–(9), Eq. (6) can be rewritten into,

$$a_2\frac{d^4v}{dx^4}-a_1\frac{d^2v}{dx^2}+a_0v=\frac{y_qq_z}{Ez_k} \tag{10}$$

Eq. (10) is a fourth-order differential equation, which, for given boundary conditions, can be solved analytically.

3. Calculation of bending stresses

The longitudinal stress at any point on the cross-section generated by the two displacement components and warping can be calculated as follows [1],

$$\sigma_x(x,y,z)=-Ey\frac{d^2v}{dx^2}-Ez\frac{d^2w}{dx^2}+E(\bar{\omega}-\omega)\frac{d^2\phi}{dx^2} \tag{11}$$

where ω is the sectorial coordinate with respect to the shear centre and $\bar{\omega}$ is the average value of ω. The first term in the right hand side of Eq. (11) is the stress generated by the deflection of the beam in horizontal direction, the second term is the stress generated by the deflection of the beam in vertical direction, and the third term is the warping stress.

Using Eq. (5) to eliminate ϕ, Eq. (11) can be rewritten into,

$$\sigma_x(x,y,z)=-Ez\frac{d^2w}{dx^2}-E\left(y+\frac{\bar{\omega}-\omega}{z_k}\right)\frac{d^2v}{dx^2} \tag{12}$$

Eq. (12) indicates that the total longitudinal stress in the beam can be decomposed into two parts. One is the stress that is generated by load q_z when the beam is considered to be fully restrained in rotation and can be calculated as follows,

$$\sigma_{x1}(x,y,z)=-Ez\frac{d^2w}{dx^2} \tag{13}$$