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Nonlinear instability of angle section beams subjected to static and dynamic sudden step loads

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ABSTRACT

This paper presents a study on the flattening behaviour of angle section beams subjected to pure bending. Analytical solutions for both static and dynamic instabilities of angle section beams subjected to pure bending about its weak axis are derived using energy methods. The results show that the dynamic instability of angle section beams under the action of a sudden step moment occurs at a moment about 71% of the corresponding critical static moment, but the deformations of the longitudinal curvature and flattening at the critical dynamic state are almost twice of those corresponding to the static instability case.

1. Introduction

Steel angle sections are widely used as columns and beams to carry axial and bending loads. Despite the apparent simplicity of the section geometry, the behaviour of angle section beams is often complicated and their strengths are difficult to predict. When an angle section beam is bent about its weak axis, Brazier's flattening deformation would occur in the section, which can lead the beam to have nonlinear snap-through instability.

From the literature review it is shown that most of existing studies focused on the linear buckling of angle section beams subjected to axial and/or bending loads. For example, Meck presented analytical solutions for buckling of a thin-walled symmetrical angle section beams loaded by a bending moment in the plane of symmetry [1]. Instability occurring either by deformation of the section or by torsional buckling was considered. Simple formulae were derived, which can be used to predict the buckling load. Wilhoit et al. [2] and Popovic et al. [3] conducted compression tests on equal angles with slender legs. The angles tested by Wilhoit et al. [2] were brake-pressed from high strength steel plates, which produced a yield strength of 465 MPa, whereas the angles tested by Popovic et al. [3] were cold-rolled and in-line galvanised, which produced a nominal yield strength of 350 MPa. A series of investigations on the performance of angle section beams subjected to various loading conditions was carried out by Earls using analytical, numerical and experimental methods [4–7]. Design recommendations were also provided by Earls [8] and by Earls and Galambos [9] for single angle flexural members. Trahair studied the behaviour of single angle steel beams. In his series of papers [10–16], the general case of unrestrained biaxial bending and torsion was simplified successively into restrained biaxial bending, lateral buckling, unrestrained biaxial bending, and buckling and torsion. Rasmussen presented an application of the direct strength method to equal angle section beam-columns with locally unstable legs [17]. In contrast to existing design methods, which independently determine the compression and bending capacities and use an interaction equation to combine these, the direct strength method determines the elastic local buckling stress for the actual stress distribution resulting from the combined action of compression and bending, and incorporates the elastic buckling stress into a direct strength equation for beam-columns. Aydin and Dogan investigated the elastic, full plastic and lateral torsional buckling problems of steel single-angle section beams subjected to biaxial bending [18]. Lately, Aydin further investigated the first yield, full plastic and critical lateral torsional buckling moments of single-angle section beams subjected to combined axial compression and biaxial bending [19]. The influence of the section flattening on the bending of the angle section beam was investigated by Kuwamura [20]. By assuming the neutral axis remains unchanged during the flattening of the section, Kuwamura managed to obtain an analytical solution for predicting the flattening behaviour of the long angle section beam under pure bending.

In this paper, the Brazier's flattening behaviour [21] of angle section beams subject to pure bending is investigated. The present

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model considers the influence of the section flattening not only on the bending rigidity but also on the position of the neutral axis. An analytical solution is derived and compared with the solution provided by Kuwamura. The static instability is further expanded to the dynamic instability case. It is well known that a structure subjected to dynamic loading may exhibit large changes in response for a small change in loading. This phenomenon is usually branded as structural dynamic instability. It is possible to categorise the problems of dynamic instability according to the types of loading and the features of the structural deformations during the instability process. Under such classification one group of problems is that where instability occurs under sudden step loads. A typical example for this case is the instability of shallow spherical shells under a sudden step pressure. Simitses developed a method of predicting critical dynamic loads for structures subjected to sudden step loads [22], in which the critical conditions are related to characteristics of the total potential energy of the system. Through this approach, the prediction of dynamic instability loads is equivalent to finding a static equilibrium point of zero potential energy. Simitses' method is particularly simple because it reduces a dynamic problem to the solution of a static problem without actually solving equations of motion. Li and Molyneaux applied Simitses' method successfully to the problem of dynamic instability of long circular cylindrical shells under pure bending [23,24]. In this paper, the Simitses' method is also applied to predict the critical dynamic moment of the long symmetrical angle section beams subjected to pure bending in the plane of symmetry.

2. Basic formulations for static analysis of instability

It is well known that when a long thin-walled circular cylindrical shell is subjected to a static pure bending, the longitudinal stresses combine with the curvature to cause a flattening of the cross-section into an oval shape [21]. This phenomenon may also occur in an angle section beam when it is bent about its weak axis.

Consider an equal leg angle section with leg length b, thickness t, and half angle α, as shown in Fig. 1a. The beam is subject to pure bending about its weak axis such that the free edge is in tension. With the increase of bending moment, the cross-section will also be flattened as shown in Fig. 1b, in which the angle plates will be bent within the beam cross-section. The total strain energy of the beam can be assumed to consist of two parts. One is the bending strain energy as a beam and the other is the bending strain energy of the two legs as the plate. These two bending strain energies can be expressed as follows,

$$U_b = \frac{1}{2} E I_w C^2 \tag{1}$$

$$U_f = 2 \times \frac{Et^3}{24(1-\nu^2)} \int_o^b \left(\frac{d^2 w}{ds^2}\right)^2 ds \tag{2}$$

where U_b and U_f are the bending strain energies of the beam and two leg plates, E is the Young's modulus of elasticity, I_w is the second moment of the flattened cross-section area, C is the longitudinal bending curvature of the beam, ν is the Poisson's ratio, w is the normal displacement of the leg plate, and s is the local coordinate as shown in Fig. 1b. Eq. (1) represents the unit length bending strain energy of a beam of infinite length with a constant bending curvature, while Eq. (2) is the unit length bending strain energy of plates due to the section flattening. The position of the natural axis and the second moment of the flattened section can be determined using the following equations,

$$\bar{y} = \frac{1}{b} \int_o^b (s\cos\alpha - w\sin\alpha) ds \tag{3}$$

$$I_w = 2t \int_o^b (s\cos\alpha - w\sin\alpha - \bar{y})^2 ds \tag{4}$$

where $\bar{y}$is the vertical distance from the neutral axis to the intersection point of the legs (see Fig. 1b). In order to calculate $\bar{y}$and I_w, the flattening shape within the section has to be assumed first. For simplicity, the flattening deformation of the leg is assumed to follow the deflection curve of a cantilever beam subjected to a moment applied at its free end, that is,

$$w(s) = w_{\max} \left(\frac{s}{b}\right)^2 \tag{5}$$

where w_{max} is the maximum deflection of the leg, which occurs at the free end. Substituting Eq. (5) into Eqs. (3) and (4), it yields,

$$\bar{y} = \frac{b\cos\alpha}{2} - \frac{w_{\max}\sin\alpha}{3} \tag{6}$$

$$I_w = bt\left(\frac{b^2\cos^2\alpha}{6} - \frac{bw_{\max}\sin 2\alpha}{6} + \frac{8w_{\max}^2\sin^2\alpha}{45}\right) \tag{7}$$

Substituting Eqs. (5)–(7) into Eqs. (1) and (2), it yields,

$$U_b = Ebt\left(\frac{b^2\cos^2\alpha}{12} - \frac{bw_{\max}\sin 2\alpha}{12} + \frac{4w_{\max}^2\sin^2\alpha}{45}\right)C^2 \tag{8}$$

$$U_f = \frac{Et^3 w_{\max}^2}{3(1-\nu^2)b^3} \tag{9}$$

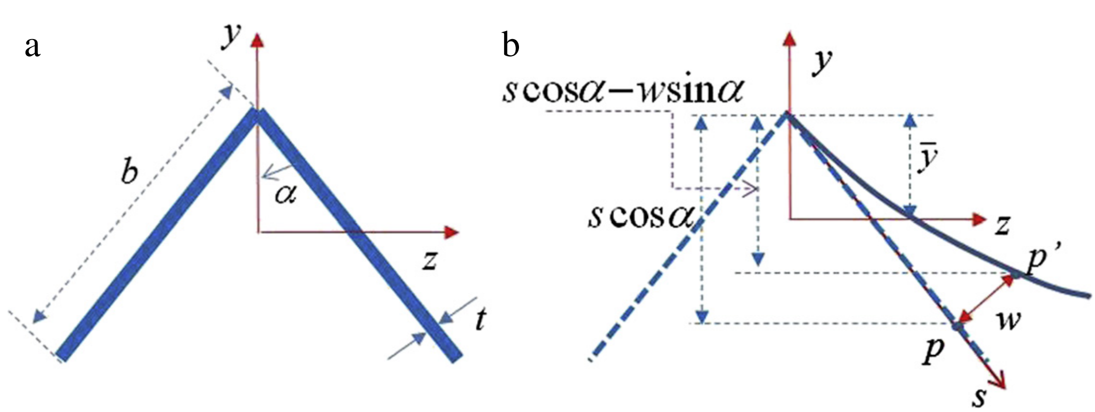

Fig. 1. (a) Angle section and (b) symbols and notations used.