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# Elastic local post-buckling of elliptical tubes

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ABSTRACT

The elastic local post-buckling behaviour of elliptical tubes under compression is analysed in this paper. A brief outline of the local, distortional and global buckling behaviour of EHS tubes is firstly provided, where it is shown that local buckling modes govern the stability of short to intermediate length tubes while distortional modes control the stability of intermediate length to moderately long tubes and global buckling dominates the behaviour of longer tubes. Following this, an in-depth numerical study employing shell finite element modelling, of the elastic local post-buckling behaviour of compressed elliptical hollow section (EHS) tubes is presented. It is concluded that EHS tubes with a low to moderate aspect ratio can support loads up to their limit loads but are imperfection sensitive (shell-type behaviour), while EHS tubes with a moderate to high aspect ratio can carry loads higher than their limit loads (plate-type behaviour) and are imperfection insensitive. The slope of the ascending post-buckling path increases with the EHS aspect ratio and can reach values up to 40% of the slope of the linear primary path. The bound imperfection amplitude concept, separating the imperfection amplitude ranges where the EHS tube is sensitive and insensitive, is proposed. It is also found that, for increasing EHS aspect ratio, the compressive stresses grow and accumulate near the zones of minimum radius of curvature while the zones of maximum radius of curvature possess an approximately uniform and relatively low compressive stress level. Therefore, it is expected that an approach based on the effective width concept widely used for the evaluation of the strength of flat plates may be adapted to the design of EHS tubes with moderate to high aspect ratios.



## 1. Introduction

The first investigation into the buckling behaviour of members with non-circular hollow sections (NCHS) under uniform compression is credited to Marguerre [1], who performed a preliminary theoretical study on the buckling mode configuration of members with an Oval Hollow Section (OHS). After that, Kempner and his collaborators made consistent investigations on the buckling and post-buckling behaviour of OHS members [2–5]. They concluded that (i) the buckling mode maximum deflection occurs at the point of maximum radius of curvature and (ii) the buckling stress of an OHS is similar to the buckling stress of an equivalent circular hollow section (CHS) with a radius equal to the maximum radius of curvature of the OHS. In addition, Feinstein et al. [4,5] observed that the local buckling stress of members with OHS depended not only on the cross-section aspect ratio $a/b$ ($2a$ and $2b$ are the major and minor axis widths) but also on the length of the member under compression. Concerning the post-buckling behaviour, Kempner and Chen [2,3] found that a decrease in the OHS thickness often leads to a more stable post-buckling behaviour and they showed that OHS members with higher aspect ratios $a/b$ possess more stable post-buckling behaviour (like flat plates) and, conversely, OHS tubes with lower aspect ratios $a/b$ exhibit more unstable post-buckling behaviour (like circular shells). Moreover, they also conclude that very eccentric OHS are much less sensitive to imperfections and the ultimate load may even be greater than the buckling load, which is in sharp contrast with the significant imperfection sensitivity of circular cylindrical shells. It was not until 1968 that the first investigation on the buckling and post-buckling behaviour of members with elliptical hollow sections (EHS) appeared. In this work, Hutchinson [6] concluded that these members are imperfection-sensitive (like circular shells) but the buckling phenomenon is not catastrophic (like in circular shells) and failure may even occur at a higher load than the buckling load. However, there was no mechanically based explanation for this evidence. A few years later, Tennyson et al. [7] also found experimentally that EHS with moderate to high eccentricity ($a/b \geq 2$) exhibited ultimate loads higher than their buckling loads. This was attributed to the fact that highly eccentric EHS display a post-buckling behaviour closer to the plate behaviour (stable) than the circular cylindrical shell one (unstable). Later, Tvergaard [8] examined the elastic–plastic post-buckling behaviour of OHS members

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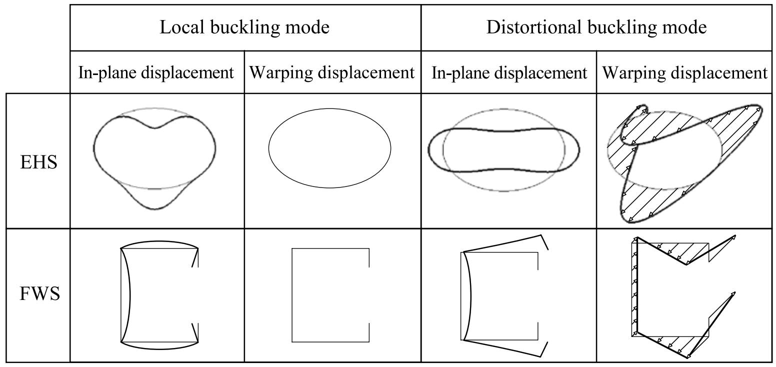


**Fig. 1.** Local and distortional buckling modes for EHS and FWS (lipped channel).

under compression and found that the spread of plasticity (yielding) near the regions of maximum radius delayed and substantially eroded the post-critical strength reserve previously unveiled by Kempner and Chen for very eccentric sections.

EHS steel tubes are now available as hot-rolled structural products [9,10] and represent an interesting solution for many visible applications in steel construction, particularly for glass facades. These shapes are included in the new edition of European Norm 10210 [11] and display a standard range of dimensions. In response to the emergence and commercial availability of EHS tubes, several recent investigations on their buckling behaviour and strength have been published. Gardner and Chan [12] and Chan and Gardner [13,14] assessed the non-linear behaviour of hot-rolled EHS tubes by means of experimental and numerical analyses and proposed structural design rules. They found that the slenderness limits for pure compression set out in EC3 for CHS classification can be safely adopted for EHS, based on the equivalent diameter of the point of the EHS with maximum radius. In addition, Zhu and Wilkinson [15] also performed shell finite element analyses to evaluate the buckling and post-buckling behaviour of EHS in compression and compared the results with the predictions obtained by means of available formulae for equivalent CHS. Roufegarinejad and Bradford [16] used an energy-based technique to investigate the local buckling of compressed EHS tubes with an elastic infill, while Yang et al. [17] and Zhao and Packer [18] studied the behaviour of concrete filled EHS in compression. Silvestre [19] developed a formulation of Generalised Beam Theory (GBT) to analyse the elastic buckling behaviour of members with NCHS and applied it to study the behaviour of EHS shells and tubes under compression, particularly the variation of the critical buckling stress with the member length and cross-section geometry. Ruiz-Teran and Gardner [20] have also examined the buckling response of EHS tubes in compression and proposed analytical formulae to accurately predict the critical stress. Therefore, the main objective of this work is to unveil the mechanics of the elastic local post-buckling behaviour of EHS tubes and to explain in a detailed fashion the transition between the shell-type (imperfection sensitive) behaviour of EHS tubes with low eccentricity and the plate-type (imperfection insensitive) behaviour of EHS tubes with high eccentricity.

## 2. Outline of EHS buckling behaviour

From previous investigations by the authors [19,20], it was found that the stability of EHS tubes is governed by three distinct buckling modes: (i) local buckling modes, (ii) distortional buckling modes and (iii) global buckling modes. Both local and distortional buckling modes are characterized by deformation of the cross-section mid-line while global buckling is associated with overall minor-axis bending of the member. Three distinct features that characterize the differences between the local and distortional buckling modes are:

(i) Local buckling modes do not exhibit primary warping along the cross-section mid-line, while distortional buckling modes do.
(ii) The stiffer parts of cross-sections do not displace in local buckling modes, while they (or at least some of them) do so in distortional buckling modes.
(iii) The half-wavelength of local buckling is approximately equal to the width of the most slender part of the cross-section while that of distortional buckling is several times greater.

The above distinctions between local and distortional buckling modes in EHS mirror those previously made for cold-formed steel members with flat-walled sections (FWS), which exhibit similar characteristics. The local and distortional buckling modes of EHS and FWS (in this case, a lipped channel section) are shown in Fig. 1. In both cases (EHS and FWS), the previous definitions apply. For instance, observe that the stiffer parts of these sections (the zones of maximum curvature in the EHS and the folds or corners of the lipped channel) do not displace in the local buckling modes. Conversely, in the distortional modes, the stiffer parts of the EHS and the lip folds of the FWS show significant displacements.

Following a number of studies of EHS, Kempner and Chen [2,3] Feinstein et al. [4,5] found that a lower bound to the EHS critical local buckling stress can be obtained from the formula to calculate the critical stress for the CHS axis-symmetric buckling mode, i.e.,

$$\sigma_{\mathrm{cr.1}} = \frac{E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{a^2/b}\right) \tag{1}$$

where the CHS radius $r$ is replaced by the maximum radius of the EHS, given by $a^2/b$ which corresponds to the least stiff regions of the EHS where the buckling lobes are located. Moreover, the half-wavelength corresponding to the critical local buckling mode is given by

$$L_{\mathrm{cr.1}} = \frac{\pi A\sqrt{t/B}}{\sqrt[4]{12(1-\nu^2)}}. \tag{2}$$

Fig. 2 shows the variation of the ratio $\sigma_{\mathrm{cr}}/\sigma_{\mathrm{cr.1}}$ with the ratio $L/L_{\mathrm{cr.1}}$ (logarithmic scale) for a typical EHS steel tube with $E = 210\,000$ N/mm$^2$ and $\nu = 0.3$ and geometrically characterised by $a = 150$ mm, $b = 100$ mm, and $t = 6$ mm. In this figure, $\sigma_{\mathrm{cr}}$ is the exact value of the EHS critical stress, $\sigma_{\mathrm{cr.1}}$ is the approximate value of the EHS critical stress calculated by means of the CHS formula (Eq. (1)), $L$ is the tube length and $L_{\mathrm{cr.1}}$ is the approximate value of the critical half-wavelength calculated by means of the CHS formula (2). From Fig. 2, it may be observed that:

(i) EHS tubes with short to intermediate lengths buckle in local critical modes, which are characterised by two buckling lobes, each one located in the minor axis region of the section, where