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Inelastic buckling of pin-ended steel columns under longitudinal non-uniform temperature distribution

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ABSTRACT

Columns under natural fire conditions are usually exposed to non-uniform temperature distribution in the longitudinal direction. The motivation for this study stems from zone modeling of a compartment fire where the gas layers are artificially divided into two zones, viz. the hotter upper zone and the cooler lower zone. However, for field modeling of a compartment fire, more detailed information of temperature distribution can be obtained. The difference in temperature between the top and bottom ends of a column can be quite significant, particularly prior to flashover condition. Depending on the required accuracy, one example due to piece-wise step distribution in the longitudinal direction is analyzed in this paper and compared with experimental results. This represents more realistically the thermal response of a column which experiences greater temperature variation with increasing height. In this paper, the inelastic stability of a pin-ended steel column under non-uniform temperature distribution is studied analytically. Across a column section, the temperature is assumed to be uniform. Two linear elastic springs connected to the column ends simulate axial restraints from adjoining unheated structures.

0. Introduction

The behavior of columns subjected to fire conditions is vastly different from that under normal ambient temperature. Thermal restraint from adjoining unheated structure plays a key role in the stability of these columns. The structural response depends largely on the temperature distribution in the cross-sectional and longitudinal directions due to (i) thermal load which will change the material properties of steel and (ii) under elevated temperature, the magnitude of thermal induced compressive stress arising from thermal restraint is of the same order as initial applied stress at ambient temperature. The effect of temperature variations in the cross-sectional direction has been studied by Ossenbruggen et al. [1]. However, this paper focuses on the analytical derivations of column stability subjected to longitudinal temperature variations since there has not been any significant theoretical development on this aspect. The objective is to derive analytical solutions to enable engineers to quickly ascertain the column stability under non-uniform temperature distribution, without recourse to numerical methods. This method is particularly useful when considering the effect of a local fire on column stability. Temperature distribution may be obtained from established fire plume models.

At present, a numerical approach is very popular in the study of structures under fire because a finite element program offers a wide range of flexibility. A review of recent literature shows that the effects of axial restraint have been investigated numerically by Neves [2], and Shepherd et al. [3]. Besides, the effects of rotational restraint have also been studied numerically by Franssen et al. [4], Wang [5] and Valente et al. [6]. It was found that the critical temperatures of columns will be reduced by axial restraint but enhanced by rotational restraint. Huang and Tan et al. [7] presented a series of numerical studies conducted on thermally-restrained steel columns subjected to predominantly axial loads. A finite element program FEMFAN3D was developed for the fire resistance analysis and creep strain has been explicitly considered. With incorporation of creep strain, the rate of increase in temperature on mechanical response can be modeled.

However, for design purposes, theoretical analysis that can be performed manually is much needed as it enables engineers to quickly ascertain the column buckling loads, particularly under local fire scenarios. Culver et al. [8] proposed an approach to determine the buckling loads for pin-ended steel columns subjected to a uniform temperature increase along the member length. In their work, the effects of residual stress and the influence of temperature on the buckling strength in both the elastic and the inelastic range were considered but thermal restraints were not taken into account. On the other hand, it is acknowledged that the behavior of a steel column in fire is mostly affected by the restraints of its adjoining structure [9]. Ali et al. [10] reported that axial restraint reduced the fire resistance of columns

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Nomenclature

A	Area of cross section of the column;
E	Young's modulus;
E_0^{20}	Elastic Young's modulus at ambient temperature;
μ	Constant factor, Young's modulus ratio;
$G_{i\alpha}$	Scalar, $i = 1, 5$;
I	Elastic bending inertia of the cross section;
I_{ep}	Inelastic bending inertia of the cross section;
k_i	Stiffness of springs, $i = 1, 2$;
k_e	Equivalent stiffness of restraints;
k_0^{20}	Axial stiffness of a column under normal ambient temperature;
L	Length of the column;
P_T	Additional axial force due to thermal expansion;
P_c	Total internal compressive axial force acting on the cross section of the column;
P_0	Service load;
P_{c-cr}	Critical total compressive axial load;
P_E	critical compressive load of an elastic pin-ended column under normal ambient temperature;
T	Temperature;
T_1	Temperature at segment 1;
T_2	Temperature at segment 2;
$\boldsymbol{\varepsilon}$	Vector, total axial strain of the column;
$\boldsymbol{\varepsilon}_{ep}$	Vector, mechanical elasto-plastic strain;
$\boldsymbol{\varepsilon}_T$	Vector, thermal induced strain;
ε	Scalar, total strain of the column;
ε_{ep}	Scalar, mechanical elasto-plastic strain;
ε_T	Scalar, thermal induced strain;
σ	Axial stress in the column;
σ_Y	Yield stress of material;
σ_{Y-1}	Yield stress of material at T_1;
σ_{Y-2}	Yield stress of material at T_2;
β	Thermal expansion ratio;
η	Ratio of critical compressive axial load;
α	Ratio of the length of segment 1 to the whole length of a column;
ϑ	Ratio of the length of segment 2 to the whole length of a column;
$\boldsymbol{\omega}$	Constant matrix;
ω_{ij}	Matrix element;
γ	Restraint stiffness ratio;

after a series of tests on 37 axially-restrained steel columns subjected to quasi-standard fire. Tang et al. [11] proposed a simple approach based on the Rankine interaction formula to obtain a realistic estimate of column fire resistance. However, in that paper, columns were subjected to uniform temperature distribution. Huang and Tan [12,13] considered the axial restraint on an isolated heated column using a linear spring attached to the column top end. They extended the traditional Rankine formula to predict the critical temperature of an axially-restrained steel column. The proposed Rankine approach incorporating both the axial restraint and creep strain yields very good agreement with the finite element predictions.

Although the stability of axially-loaded columns at elevated temperature and subjected to elastic restraints has been studied by some researchers, the assumption about uniform temperature distribution may give an overly conservative prediction, since the temperature distribution in the longitudinal direction under fire conditions is usually non-uniform. This is because through the convective process, the hottest layer of air will rise up to the top with a relatively cooler layer at the bottom. Thus, based on uniform temperature assumption, for conservatism, engineers usually ascribe the hottest temperature at the column top as the uniform column design temperature. In 1972, Culver [14] analyzed the stability of wide-flanged steel columns subject to elevated temperature using a finite difference approach. The buckling loads were determined by solving the governing differential equation based on the finite difference method. Various cases of non-uniform temperature distribution along the member length were considered, but the influence of end restraints was not investigated. In this paper, as shown in Fig. 1, the temperature in the longitudinal direction (x-axis) is assumed to be non-uniform. Two linear springs attached to the column ends simulate the linear restraints from the adjoining unheated structure. To simplify the ensuing derivations, the two elastic springs can be replaced by one equivalent spring (k_e) at the top end of the column. The critical load is derived analytically using Galerkin's method. Tan and Yuan [15] studied the stability of a pin-ended steel column subjected to varying longitudinal temperature distribution but the analysis was only conducted for an elastic material model. In this paper, the inelastic buckling load is derived analytically.

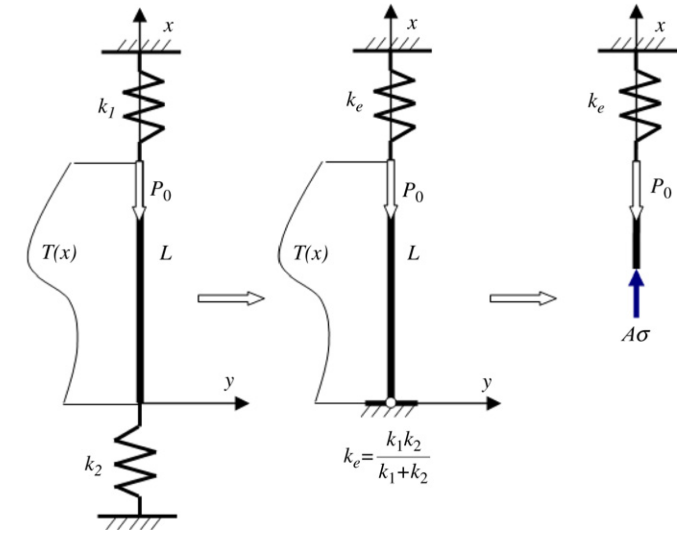

Fig. 1. Column member under compressive load.

1. Stability

1.1. Stress in the column

For a pin–pin ended column under non-uniform temperature distribution, Young's modulus of steel is no longer constant, since temperature varies over the column height. Adopting the coordinate system shown in Fig. 1, Young's modulus can be expressed by (1):

$$E = E(T) = E(x). \tag{1}$$

With the ends restrained by linear springs and without any imposed external load, the total axial strain of a heated column can be described by (2):

$$\varepsilon = -\varepsilon_{ep} + \varepsilon_T \tag{2}$$

where ε is the positive column strain in tension. The term ε_{ep} is the mechanical elasto-plastic strain and ε_T is the thermally induced strain. In (2), it is assumed that the axial force is compressive and the temperature is elevated.