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Nonlinear inelastic analysis of space frames

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ABSTRACT

In this paper, a fiber beam–column element which considers both geometric and material nonlinearities is presented. The geometric nonlinearities are captured using stability functions obtained from the exact stability solution of a beam–column subjected to axial force and bending moments. The material nonlinearities are included by tracing the uniaxial stress–strain relationship of each fiber on the cross sections. The nonlinear equilibrium equations are solved using an incremental iterative scheme based on the generalized displacement control method. Using only one element per member in structure modeling, the nonlinear responses predicted by the proposed element compare well with those given by commercial finite element packages and other available results. Numerical examples are presented to verify the accuracy and efficiency of the proposed element.

1. Introduction

In the past few decades, there have been numerous studies to improve the accuracy of the beam–column element for the nonlinear analysis of steel frames. In general, the nonlinear response of steel frames can be predicted by using either the finite element method or the beam–column approach. The finite element approach is often based on a stiffness or displacement formulation in which cubic and linear interpolation functions are used for the transverse and axial displacements, respectively [1–5]. Since this method is based commonly on an assumed cubic polynomial variation of transverse displacement along the element length, it is unable to capture accurately the effect of axial force acting through the lateral displacement of the element (P–δ effect) when one element per member is used [6]. Hence, it overestimates the strength of a member under significant axial force. Although the accuracy of this method can be improved by using several elements per member in the modeling, it is generally recognized to be computationally intensive because of a very refined discretisation of the structures. The beam–column approach is based on the stability functions which are derived from the exact stability function of a beam–column subjected to axial force and bending moments [7–12]. This approach can capture accurately the P–δ effect of a beam–column member by using only one or two elements per member in the modeling, hence, to save computational time.

In parallel with the above developments, different beam–column models have been proposed to represent inelastic material behavior. These models can be grouped into two categories: lumped plasticity [9,10,13] model and distributed plasticity model [5,14–18]. In the lumped plasticity model, the inelastic behavior of material is assumed to be concentrated at point hinges that are usually located at the ends of the member. The force–deformation relation at these hinges is based on force resultants. The advantage of this model is that it is simple in formulation as well as implementation. However, the disadvantage of this model is that the force–deformation relation at the hinges is not always available and accurate for every section. In the distributed plasticity model, the inelastic behavior of material is distributed along the member length since the element behavior is monitored through numerical integration of constitutive behavior at a finite number of control sections. The nonlinear constitutive behavior at these sections is derived using one of the following methods: (1) moment–curvature relations; (2) force and deformation resultants; and (3) uniaxial stress–strain relations of fibers on the cross sections. Although fiber model is the most computationally intensive among others, it represents the inelastic behavior of material more accurately and rationally than concentrated plasticity model.

This paper proposes a fiber beam–column element for the nonlinear inelastic analysis of space steel frames. The spread of plasticity over the cross section and along the member length is captured by tracing the uniaxial stress–strain relations of each fiber on the cross sections located at the selected integration points along the member length. The Gauss–Lobatto integration rule is adopted herein for evaluating numerically element stiffness matrix instead of the classical Gauss integration rule because it always includes the end sections of the integration field. Since inelastic behavior in beam elements often concentrates at the ends of the member, monitoring the end sections of the element results in improved accuracy and numerical stability [19].

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Although the fiber model is included in DRAIN-3DX [20] and OpenSees [21] programs to represent the material nonlinearity, the geometric nonlinearity caused by the interaction between the axial force and bending moments (P–δ effect) was not considered. Therefore, these methods overestimate the strength of a member subjected to significant axial force if only one or few elements per member are used in the modeling. In this research, the stability functions obtained from the closed-form solution of a beam–column subjected to end forces are used to accurately capture the P–δ effect. Numerical examples are presented to verify the accuracy and efficiency of the proposed element in predicting nonlinear inelastic response of space steel frames.

2. Element formulations

2.1. Geometric nonlinear P–δ effect

To capture the effect of axial force acting through the lateral displacement of the beam–column element (P–δ effect), the stability functions reported by Chen and Lui [22] are used to minimize modeling and solution time. Generally only one element per member is needed to accurately capture the P–δ effect. From Kim et al. [10], the incremental force–displacement equation of space beam–column element which accounts for transverse shear deformation effects can be expressed as

$$\begin{Bmatrix} \Delta P \\ \Delta M_{yA} \\ \Delta M_{yB} \\ \Delta M_{zA} \\ \Delta M_{zB} \\ \Delta T \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{1y} & C_{2y} & 0 & 0 & 0 \\ 0 & C_{2y} & C_{1y} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{1z} & C_{2z} & 0 \\ 0 & 0 & 0 & C_{2z} & C_{1z} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} \end{bmatrix} \times \begin{Bmatrix} \Delta \delta \\ \Delta \theta_{yA} \\ \Delta \theta_{yB} \\ \Delta \theta_{zA} \\ \Delta \theta_{zB} \\ \Delta \phi \end{Bmatrix} \tag{1}$$

where ΔP, ΔM_{yA}, ΔM_{yB}, ΔM_{zA}, ΔM_{zB}, and ΔT are incremental axial force, end moments with respect to y and z axes, and torsion respectively; $\Delta\delta$, $\Delta\theta_{yA}$, $\Delta\theta_{yB}$, $\Delta\theta_{zA}$, $\Delta\theta_{zB}$, and $\Delta\phi$ are the incremental axial displacement, joint rotations, and angle of twist; C_{1y}, C_{2y}, C_{1z}, and C_{2z} are bending stiffness coefficients accounting for the transverse shear deformation effects, and are expressed as

$$C_{1y} = \frac{k_{1y}^2 - k_{2y}^2 + k_{1y}A_{sz}GL}{2k_{1y} + 2k_{2y} + A_{sz}GL} \tag{2a}$$

$$C_{2y} = \frac{-k_{1y}^2 + k_{2y}^2 + k_{2y}A_{sz}GL}{2k_{1y} + 2k_{2y} + A_{sz}GL} \tag{2b}$$

$$C_{1z} = \frac{k_{1z}^2 - k_{2z}^2 + k_{1z}A_{sy}GL}{2k_{1z} + 2k_{2z} + A_{sy}GL} \tag{2c}$$

$$C_{2z} = \frac{-k_{1z}^2 + k_{2z}^2 + k_{2y}A_{sy}GL}{2k_{1z} + 2k_{2z} + A_{sy}GL} \tag{2d}$$

where $k_{1n} = S_{1n}(EI_n/L)$ and $k_{2n} = S_{2n}(EI_n/L)$; S_{1n} and S_{2n} are stability functions with respect to n axis ($n = y, z$), and are expressed as

$$S_{1n} = \begin{cases} \dfrac{k_nL\sin(k_nL) - (k_nL)^2\cos(k_nL)}{2 - 2\cos(k_nL) - k_nL\sin(k_nL)} & \text{if } P < 0 \\[2ex] \dfrac{(k_nL)^2\cosh(k_nL) - k_nL\sinh(k_nL)}{2 - 2\cosh(k_nL) + k_nL\sinh(k_nL)} & \text{if } P > 0 \end{cases} \tag{3a}$$

$$S_{2n} = \begin{cases} \dfrac{(k_nL)^2 - k_nL\sin(k_nL)}{2 - 2\cos(k_nL) - k_nL\sin(k_nL)} & \text{if } P < 0 \\[2ex] \dfrac{k_nL\sin(k_nL) - (k_nL)^2}{2 - 2\cosh(k_nL) + k_nL\sinh(k_nL)} & \text{if } P > 0 \end{cases} \tag{3b}$$

where $k_n^2 = |P|/EI_n$. EA, EI_n, and GJ denote the axial, bending and torsional stiffness of the beam–column element, and are defined as

$$EA = \sum_{j=1}^{h} w_j \left(\sum_{i=1}^{m} E_iA_i \right)_j \tag{4}$$

$$EI_y = \sum_{j=1}^{h} w_j \left(\sum_{i=1}^{m} E_iA_iz_i^2 \right)_j \tag{5}$$

$$EI_z = \sum_{j=1}^{h} w_j \left(\sum_{i=1}^{m} E_iA_iy_i^2 \right)_j \tag{6}$$

$$GJ = \sum_{j=1}^{h} Gw_j \left[\sum_{i=1}^{m} (y_i^2 + z_i^2)A_i \right]_j \tag{7}$$

in which h is the total number of monitored sections along an element; m is the total number of fibers divided on the monitored cross section; w_j is the weighting factor of the jth section; E_i and A_i are the tangent modulus of the material and the area of ith fiber, respectively; y_i and z_i are the coordinates of ith fiber in the cross section. The element force–deformation relationship of Eq. (1) can be expressed in symbolic form as

$$\{\Delta F\} = [K_e]\{\Delta d\} \tag{8}$$

where

$$\{\Delta F\} = \begin{bmatrix} \Delta P & \Delta M_{yA} & \Delta M_{yB} & \Delta M_{zA} & \Delta M_{zB} & \Delta T \end{bmatrix}^T \tag{9}$$

$$\{\Delta d\} = \begin{bmatrix} \Delta \delta & \Delta \theta_{yA} & \Delta \theta_{yB} & \Delta \theta_{zA} & \Delta \theta_{zB} & \Delta \phi \end{bmatrix}^T. \tag{10}$$

The element stiffness matrix is evaluated numerically by the Gauss–Lobatto integration scheme since this method allows for two integration points to coincide with the end sections of the elements [23]. Since inelastic behavior in beam elements often concentrates at the ends of the member, the monitoring of the end sections of the element is advantageous from the standpoint of accuracy and numerical stability. By contrast, the outermost integration points of the classical Gauss integration method only approach the end sections with increasing order of integration, but never coincide with the end sections and, hence, result in overestimation of the member strength [24].

2.2. Material nonlinear effect

In order to capture the gradual plastification throughout the member's cross section, a fiber model as shown in Fig. 1 is used. The fiber beam–column element is divided into a discrete number of monitored sections represented by the integration points. Each monitored section is divided into m fibers and each fiber is represented by its area A_i and coordinate location corresponding to its centroid (y_i, z_i). Section deformations are represented by three strain resultants: the axial strain ε along the longitudinal axis and two curvatures χ_z and χ_y with respect to z and y axes, respectively. The corresponding force resultants are the axial force N and two bending moments M_z and M_y. The section forces and deformations are grouped in the following vectors:

$$\text{Section force vector } \{Q\} = \begin{bmatrix} M_z & M_y & N \end{bmatrix}^T \tag{11}$$

$$\text{Section deformation vector } \{q\} = \begin{bmatrix} \chi_z & \chi_y & \varepsilon \end{bmatrix}^T. \tag{12}$$