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Buckling analysis of a cable-stayed circular frame

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ABSTRACT

Large parabolic dish concentrators have been widely employed in solar thermal applications. The supporting structure of a solar dish concentrator consists of a circular frame, a central post, and front and rear cables connecting the frame to the post. The tensions in the cables cause compressive stresses in the circular frame and the central post, and this support structure must be designed for stability. In this paper, the nonlinear buckling behavior of the supporting structure of a cable-stayed circular frame is studied in detail. A three-dimensional finite element model of the supporting structure is developed to predict the critical cable tensions that would cause buckling of the circular frame and to determine the associated buckling mode shapes of the supporting structural system. The results show that the buckling load of a cable-stayed circular frame depends not only upon the cross-section of the frame, but also upon the number of cables and the inclinations of the cables. In all cases, in-plane buckling modes are predominant. The concentrated torques resulting from unbalanced cable tensions tend to induce the out-of-plane buckling modes.

1. Introduction

Large parabolic dish concentrators, such as shown in Fig. 1, have been widely employed in solar thermal applications [1] to reflect sunlight to a focal point where solar radiation is collected for energy conversion. Common solar dish concentrators have a focal length to diameter ratio of about 0.6. Reflector surfaces of high optical quality are attainable with the use of metallic [2] or fiber-reinforced composite membranes. The supporting structure of such solar dish concentrators has an appearance similar to a bicycle wheel hub (see Fig. 2), consisting of a circular frame, a central post, and front and rear cables connecting the frame to the post. The circular frame has a box cross-section. The cables are attached to the inner edge of the circular frame at equidistant locations along its circumference. The cables are pretensioned to provide stiffness for this basic structural configuration. The tensions in the cables cause compressive stresses in the frame and the central post, and consequently the frame and the central post must be designed to meet stability requirements.

This paper investigates the buckling behavior of the supporting structure of a solar dish concentrator. The diameter of the circular frame in this study is 15 m, and the focal length of the collector is 9 m. The design variables include the geometry and materials of the supporting frame, the number of cables, the cable pretensions, the inclination of the rear cables, and the dimension of the central post.

Nonlinear buckling analyses using finite element models of various geometric frame configurations are conducted to optimize this cable-stayed frame design for the solar dish concentrator. Effects such as those caused by initial imperfection of the circular frame construction and uneven cable pretensions are not considered in the analysis.

2. Three-dimensional deformations of a circular frame

The three-dimensional deformations of a differential element of arc length $Rd\theta$ from a circular frame are shown in Fig. 3(a), where R is the mean radius of the circular frame. The circumferential, axial, and radial displacements of the centroidal axis are denoted by u, v, and w, respectively. These displacements are assumed to be along the positive x, y, and z coordinate axes, whose origin is located at the centroid of the cross-section. The angle of roll at a section of the frame is denoted by ϕ, as shown in Fig. 3(b).

The in-plane bending moment can be expressed as [3]

$$M_y = \frac{EI_y}{R^2}\left(\frac{d^2 w}{d\theta^2} + w\right). \quad (1)$$

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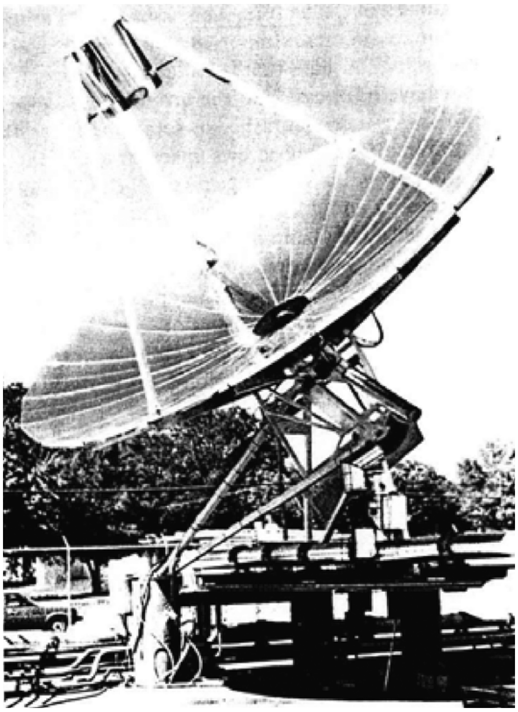

Fig. 1. Parabolic solar dish concentrator installed at Shenandoah, GA. (Courtesy of Sandia National Laboratories.)

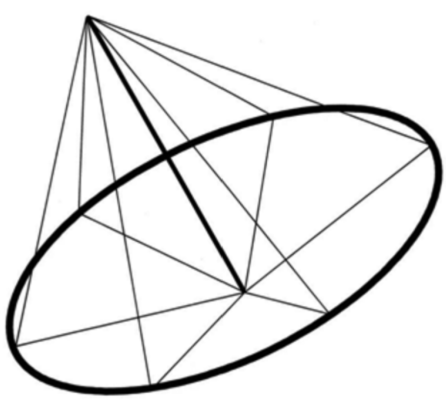

Fig. 2. Supporting structure of a solar dish concentrator.

Note that the in-plane bending is not coupled with the circumferential or the axial displacement components. The out-of-plane bending moment can be expressed as [4]

$$M_z = \frac{EI_Z}{R^2}\left(\frac{d^2 v}{d\theta^2} - R\phi\right) \quad (2)$$

and the torsional moment in the frame is written as

$$T_X = \frac{GK}{R}\left(\frac{d\phi}{d\theta} + \frac{1}{R}\frac{dv}{d\theta}\right). \quad (3)$$

In the above equations, EI_y is the in-plane flexural rigidity, EI_z is the out-of-plane flexural rigidity, and GK is the torsional rigidity of the circular frame. The positive senses of the in-plane and out-of-plane forces and moments are shown in Fig. 4.

Meek [5] investigated the buckling of circular frames of arbitrary sections and showed that the in-plane and out-of-plane deformations are coupled due to a nonzero product of inertia of the section. Seide and Albano [6] pointed out that it is the rotation of the section of a circular frame that gives rise to the instability, the so-called "toroidal buckling", even though the frame might be loaded in its plane. Kim and Suh [7] studied the spatial stability of curved beams with nonsymmetric cross-section based on the displacement field. Other researchers [8–11] also investigated the in-plane and out-of-plane buckling as well as flexural–torsional buckling for steel arched beams. In the case of a cable-stayed circular frame, the in-plane and out-of-plane deformations are coupled through the compatibility requirements imposed by the cable deformations, regardless of whether the frame cross-section is doubly symmetric or not.

3. Equations of equilibrium in the deformed state

Based on the forces and moments acting on a circular frame element, as shown in Fig. 4, the following equations of equilibrium can be derived.

3.1. Equilibrium of forces

Equilibrium of the forces in the circumferential direction gives

$$\frac{dN}{d\theta} - H = 0. \quad (4)$$

Equilibrium of the forces in the radial direction gives

$$\frac{dH}{d\theta} + N - \frac{N}{R}\left(\frac{d^2 w}{d\theta^2} + w\right) = 0. \quad (5)$$

Equilibrium of the forces in the axial direction gives

$$V_y = \frac{N}{R}\frac{dv}{d\theta} \quad (6)$$

if the applied radial loads remain parallel to the plane of the circular frame.

3.2. Equilibrium of moments

Equilibrium of the in-plane moments gives

$$\frac{1}{R}\frac{dM_y}{d\theta} + H = 0. \quad (7)$$

Equilibrium of the out-of-plane moments gives

$$\frac{1}{R}\left(\frac{dM_z}{d\theta} - T_x\right) + V_y = 0. \quad (8)$$

Equilibrium of the twisting moments gives

$$\frac{dT_x}{d\theta} + M_z = 0. \quad (9)$$

Eqs. (4), (5) and (7) can be combined into a single equation of in-plane equilibrium as

$$\frac{d}{d\theta}\left(\frac{d^2 M_y}{d\theta^2} + M_y\right) = -N\frac{d}{d\theta}\left(\frac{d^2 w}{d\theta^2} + w\right) - \frac{dN}{d\theta}\left(\frac{d^2 w}{d\theta^2} + w\right). \quad (10)$$

In addition, Eqs. (6) and (8) can be combined into one equation as

$$\frac{dM_z}{d\theta} - T_x + N\frac{dv}{d\theta} = 0. \quad (11)$$

4. Buckling of a circular frame under concentrated radial forces

The theoretical buckling loads of a circular frame under concentrated radial forces and twisting moments may be used for preliminary design, in which the stiffness contribution from the