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Mechanical model for the analysis of steel frames with semi rigid joints

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ABSTRACT

The rigidity of joints is known to affect the structural behaviour of steel frames. Accurate determination of such rigidity may require the use of laborious numerical modeling (such as Finite Element) of the joint. The main objective of this paper is to present a mechanical model in order to take into account the influence of the joints on the behaviour of steel frames. This mechanical model is based on the analogy of three springs, and a non deformable element of nodes describing relative displacements and rotations between the nodes and the elements of the structure. For this model, a stiffness matrix and a nodal load vector of a beam element in bending are obtained. Examples are provided to illustrate the simplicity and efficiency of the method.

1. Introduction

In the traditional analysis and design of steel structures, frames are analysed and designed under the simplifications that the connections behave either as pinned or rigid. The use of an ideally pinned condition implies that no moment will be transmitted from beam to column. The fully rigid condition implies that no rotation occurs between the joining members [1,2]. However, these two cases of behaviour are extreme as most connections used in common practice transmit some partial moment.

To assess the real behaviour of the frame, it is therefore necessary to incorporate the effect of connection flexibility of the frame [2–6]. The flexibility of connections depend on the deformation of the fasteners (bolts, end plate, angle flange cleats, etc.), the type of connections, their position and the local deformation of the assembled elements [7–9].

Since the connection details consist of member components, any change in these connection details may lead to significant variations in the connection characteristics [10–12].

Some researchers such as Kishi and Chen [9], have collected available experimental results and constructed steel connection data banks that provided the user with not only the test data, but also some predictive equations. However, not every structural engineer has access to the database of experimental results. Also, when the connections detailing, beam and column sizes used in frame analysis are significantly different from the available experiments, however, the connection behaviour retrieved from a database may not, correctly, represent the actual connections.

De Lima et al. [13] used the concept of neural networks to determine the initial stiffness of beam-to-column joints. However, the method was limited in scope and the authors did not back their results with test data to validate the method developed. Lopez et al. [14] developed a model that takes into account the rigidity of the joints in the analysis of single-layer lattice domes. The model is based on both numerical model and test results. Del Savio et al. [15] developed a model based on a parametric study of semi-rigid joints used for the analysis of Vierendeel girders.

Experimental results [1,7,8,10–13,16] obtained for beam-column connections show that the moment–rotation relationship is non-linear for all types of connections and varies depending on connection flexibility. It is presented in its exponential form [17,18] by the equation:

$$\Theta = kM^{\alpha}. \qquad (1)$$

Because of the high number of the parameters influencing the behaviour of connections, accurate modeling of such behaviour becomes complex. Globally, initial rigidity and the ultimate moment of the connection are the two most significant characteristics to define the behaviour of a joint [2,17,18].

2. Mechanical model

The adopted model [17] is based on the analogy of three springs (two translational and one rotational) by considering the concept of a non-deformable element of node describing relative displacements and rotations between the nodes and the elements of the structure.

The nodes of the structure in Fig. 1(a) are represented by a non deformable frame as in the Fig. 1(b) where the nodes are modeled as translational and rotational springs connected to the

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Notations

k_c; k_s	Secant rigidity and secant flexibility of the connection, respectively (functions of rotation Θ and moment M)
k_1, k_2	Elastic constants of the springs in rotation at nodes "i" and "j", respectively
$k^{(i)}$	Flexibility in stage "i"
nl, ml	Distance to left support and right support respectively, from gravity centre Ψ
ω	Flexural rigidity per unit length, $\frac{EI}{l}$
Δ_i	Relative vertical displacement between nodes "i" and "j"
V_i, M_i, V_j, M_j	Reactions at nodes "i" and "j", in local reference.
$\overline{F_e}$	Vector force in local reference.
Ψ	Area of bending moment diagram for a simply supported beam.
$\Delta W^{(i)}$	Increment of loads at stage "i"

bar element (see Fig. 1(c)). Thus, the ends of the bar element possess relative displacements and relative rotations.

The objective of the mechanical model is to derive in a simple way, both the stiffness matrix and the load nodal vector. For this, the bar element subjected to transversal loads Fig. 2(a) with semi-rigid joints (Fig. 2(b)) is considered.

2.1. Equilibrium equations and rotational deformations

The equilibrium equations may be written as:

$$V_i + V_j - R = 0 \tag{2a}$$

$$M_i + M_j + RZ - V_j l = 0. \tag{2b}$$

In bending, the rotational spring is the essential component and hence the equations of rotational deformations can be expressed as:

$$\Theta_i = \frac{\Delta_i}{l} + \frac{m\Psi}{\omega l} + \frac{M_i}{3\omega} + k_1 M_i^{\alpha} - \frac{M_j}{6\omega} \tag{3a}$$

$$\Theta_j = \frac{\Delta_i}{l} - \frac{n\Psi}{\omega l} + \frac{M_j}{3\omega} + k_2 M_j^{\alpha} - \frac{M_i}{6\omega}. \tag{3b}$$

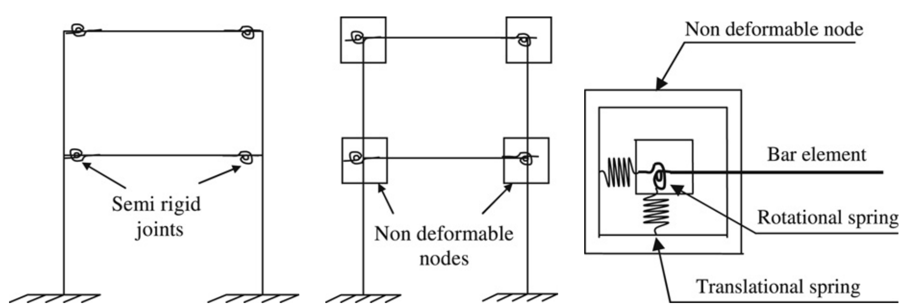
2.2. Stiffness matrix

The displacement method, which is based on a stiffness matrix, is used to analyse the frame elements.

To establish the modified stiffness matrix considering the effect of connection flexibility, the direct method is used, i.e. the rigidity k_{ij} of an element "ij" is the reaction in the direction "j" due to a unit displacement in the direction "i".

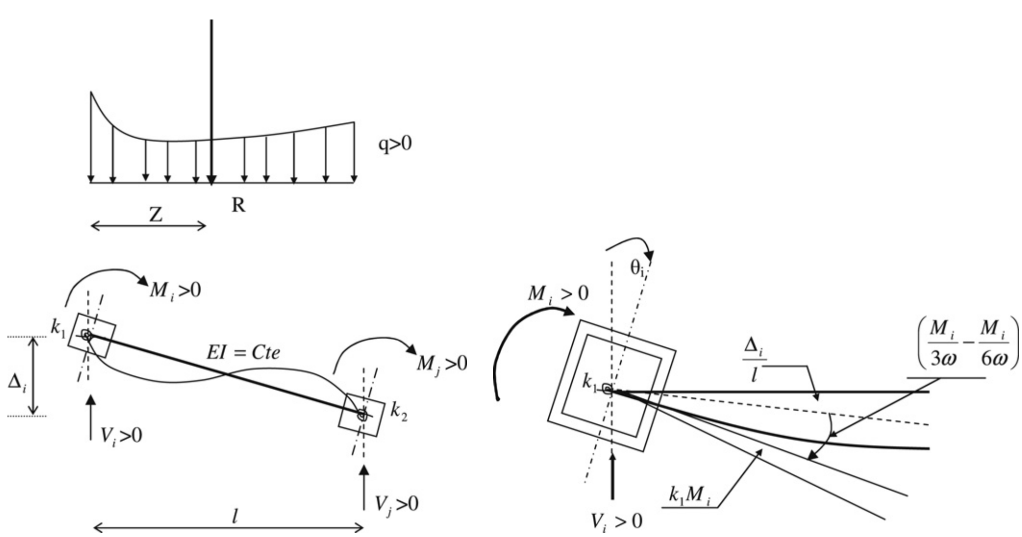
The stiffness matrix, $\overline{K_e}$, in local coordinates is given by:

$$\overline{K_e} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix}. \tag{4}$$



(a) Semi rigid joints. (b) Non deformable node. (c) Bar element and non deformable node.

Fig. 1. Mechanical model adopted.



(a) Bar element under transversal loads. (b) Different rotations of the node "i".

Fig. 2. Non-deformable node with semi-rigid joints.