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Buckling analysis of structural steel frames with inelastic effects according to codes

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ABSTRACT

Steel building frames are often analyzed for stability in an elastic way, while most of their columns behave inelastically at the buckling stage. Most column design provisions allow for inelastic behavior, but overall inelastic stability analysis is rarely performed. In this study the analysis philosophy is centered on the overall frame stability and its true safety factor. As many columns show inelastic behavior at the buckling stage, the proposed procedure takes due consideration of this fact. Once the overall buckling factor for the frame is obtained, individual column effective length factors, and their true slenderness ratios are computed, and used in the design relationships. This procedure circumvents the use of design nomographs and numerous formulas proposed in the past to alleviate their shortcomings. The procedure proposed based on the overall safety factor concept is an iterative one. It starts with a stability analysis and gradually modifies the structural properties to take account of inelasticity and eventually converges to the final buckling factor and mode shape. Any type of lateral restraint can be exactly modeled and taken into consideration without the need for approximate simplifying assumptions. The design philosophy proposed herein is that all columns must have their design parameters as related to buckling capacity must be derived from a single buckling analysis valid for the whole structure, and not considered separately and isolated from the rest of the structure as is currently practiced. Examples are worked out to illustrate the procedure and the results are compared to those of others.

1. Introduction

Historically inelastic buckling of steel columns had always been of interest to investigators in the field of structural steel work. Engesser et al. had noticed the deviation of the column buckling curve from the Euler's theoretical one in all the early tests performed. The background of investigations in this regard are given in most books on the subject and are not repeated here [1,2]. It turns out that most columns due to their residual stresses resulting from mill rolling, fabrication, etc, behave inelastically at the buckling stage. However, in the design process of such frames, the analysis is often performed as completely elastic but in the member sizing stage the inelasticity effects are taken into account. Such inelastic behavior is usually specified by the respective code of practice in a simplified way. It is therefore, logical that such inelastic effects be considered from the outset in the analysis stage. A procedure is proposed here that enables the designers to accomplish this task.

Elastic buckling of a column pin ended of length L, with flexural stiffness property of EI, is expressed in Euler's equation (1) below.

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}. \tag{1}$$

The corresponding critical stress can be expressed as in Eq. (2), in which the length factor K takes into account any other boundary conditions [3].

$$F_{cr} = \frac{\pi^2 E}{(KL/r)^2}. \tag{2}$$

Therefore in Eq. (2), the slenderness ratio KL/r, becomes the main variable in determination of the critical buckling stress.

Results of numerous tests to determine the actual critical stresses, showed deviations from the above theoretical curve, when the slenderness ratio KL/r falls below certain limits, as shown in Fig. 1. The pattern of scatter and the general trend of the deviated test points, depend on several factors and would differ for different types of column sections. However, early in time a conclusion became definite that Euler curve could not be depended upon for design in all ranges of slenderness ratios. In fact it turned out that most economically designed columns fall below the so called elastic range of the buckling curve. The reasons for this inelastic behavior are explained in detail in most textbooks on the subject, as mainly due to the entrapment of thermal residual stresses [1,4,5].

In order to open the way for designers to have simple formulas for column design in the whole range of slenderness ratios; early

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Nomenclature

A	Column cross section area
C_c	Slenderness ratio at boundary of inelastic range
$\{D\}$	Nodal displacement vector
E	Modulus of elasticity
E_e	Effective elastic modulus
F_{cr}	Column critical axial stress
F_{pl}	Stress at proportional limit
F_y	Yield stress
I	Column moment of inertia
$[K_f]$	Stiffness matrix
K	Column effective length factor
L	Column length
KL/r	Column slenderness ratio
P_{cr}	Column critical axial force
P	Column axial force
r	Column radius of gyration
RE	Ratio of effective over elastic moduli
$[S_f]$	Stability matrix
λ	Eigen value or buckling factor
k_{br}	Bracing spring constant
$[U]$	Upper triangular matrix
$[\Gamma]$	Diagonal matrix

in the 1960's, the CRC (Column Research Council), now known as the SSRC, came up with a parabolic curve to cover the range of the inelastic KL/r values in the critical stress curve. This equation which is only based on empirical results, was made tangent to the Euler's elastic curve at a point whose slenderness would produce half of the yield stress on the elastic curve, reaching the summit at yield stress on the vertical axis, as shown in Fig. 2. Thus the following AISC (American Institute for Steel Construction) design equations to cover the full range of slenderness ratios emerged for column design [2,6].

$$F_{cr} = \frac{\pi^2 E}{(KL/r)^2} \quad \frac{KL}{r} \geq C_c \tag{3}$$

$$F_{cr} = F_y\left(1 - \frac{1}{2}\left(\frac{KL/r}{C_c}\right)^2\right) \quad \frac{KL}{r} \leq C_c \qquad C_c = \pi\sqrt{2E/F_y}. \tag{4}$$

It is noteworthy, that in some other countries other forms of equations have been used to cover the same inelastic buckling range, possibly to suit the transition equation better to their local data from their structural steel practice [2,4].

From the above design equations it is clear that the correct value of the effective length factor K becomes a critical parameter in the design steps. AISC Code [6] originally advocated the use of nomographs based on the assumption that the structural frame is either fully braced laterally at each floor, or has no lateral bracings, and is free to sway, and also based on some simplifying assumptions regarding the frame configuration. Currently none of the original assumptions is considered realistic and sufficient, as most practical cases do not satisfy those conditions exactly, hence violate the simplifying assumptions. This situation has prompted many to try to offer remedies at the same time abiding by the code provisions. In the last few decades, many investigators such as Chen [7], Bridge and Fraser [8], Chu and Chow [9], LeMessurier [10], Canle [11], Yura [12], Disque [13] and several others [14–18], have tried to supplement the so-called nomograph approach with factors to come up with more realistic results. However, despite all the efforts made, the fact remains that the K factor computed by these methods, at best is locally determined for each column, mindless of the fact that the buckling of each column does not occur independently, but as part of the whole frame. When the frame buckles, it could be as the result of the buckling of a single column or as a result of a combination or a group of members buckling. It remains to be found which configuration of buckling constitutes the weakest link corresponding to the lowest buckling load factor. Therefore, an overall buckling analysis is required and any K factor based on buckling assumptions unrelated to those results would be unrealistic, and could be grossly in error. Attempts to find the overall buckling capacity of the frame by a tedious non-linear incremental analysis following a finite element procedure were conducted mostly as research aimed at finding some insight into the actual inelastic behavior of frames. They were not intended as a standard design procedure as referred to in [19], and are considered as impractical in actual design cases. It must be pointed out that other flexural members in a framework such as girders do not exhibit such inelasticity at the frame buckling stage and can be treated as elastic members.

Therefore, in this investigation based on [20], a general routine for the stability analysis of the overall framework is proposed and discussed which takes into account the inelasticity of the column members by iterative cycles to convergence to their true slenderness, and final emergence of a unique stability load factor.

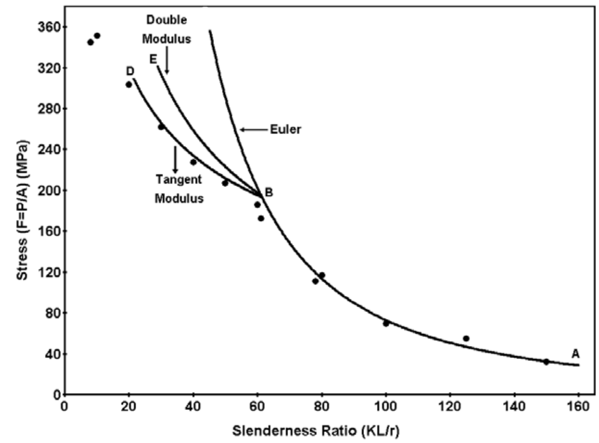

Fig. 1. Typical comparison of theoretical and experimental capacities of columns.

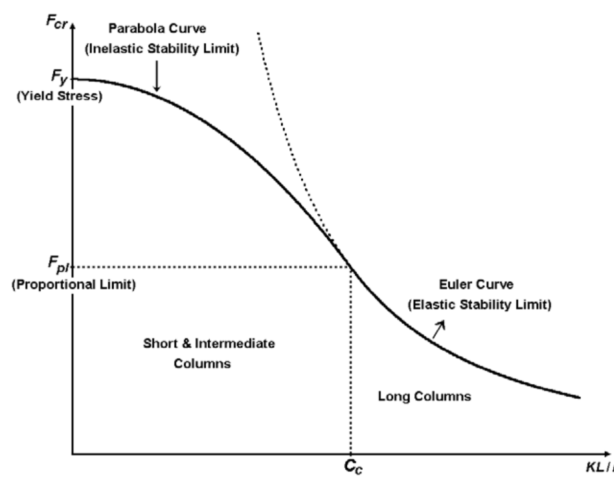

Fig. 2. Critical column stress variation vs. slenderness ratio based on experimental data.